Nonlinearities and Parameter Heterogeneity in the Determinants of Economic Growth: An Application of Nonparametric Model Selection

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Abstract

Recent research on macroeconomic growth has been focused on resolving several key issues, two of which, specification uncertainty of the growth process and variable uncertainty, have received much attention in the recent literature. The standard procedure has been to assume a linear growth process and then to proceed with investigating the relevant variables that determine growth across countries. However, a more appropriate approach would be to recognize that a misspecified model may lead one to conclude that a variable is relevant when in fact it is not. This paper takes a step in this direction by considering conditional variable uncertainty with full blown specification uncertainty. We use recently developed nonparametric model selection techniques to deal with nonlinearities and competing growth theories. We show how one can interpret our results and use them to motivate more intriguing specifications within the traditional studies that use Bayesian Model Averaging or other model selection criteria. We find that the inclusion of nonlinearities is important for determining the empirically relevant variables that dictate growth

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and that nonlinearities are especially important in uncovering important features of the growth process.

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"Heterogeneity is a key feature of national experiences so that even if one is willing to consider a common model for uniting these experiences, the parameters of the model are very likely to differ across countries." Durlauf (2000 pg. 13)

1 Introduction

Recently much attention in the growth empirics literature has paid attention to four key tenets: (i) parameter heterogeneity, (ii) the consequences of competing economic growth theories, (iii) nonlinearities in the growth process and (iv) association versus causation.[1] Our aim is to address tenets (i), (ii) and (iii) in a near simultaneous fashion. This is empirically interesting as it can be seen as a step towards motivating more appropriate parametric specifications for use with Bayesian model averaging (BMA) and other model selection methods currently used to investigate competing growth theories.

As it stands, point (ii) can be subdivided into further categories, two of which can be classified as variable uncertainty and specification uncertainty. Typically, model selection studies assume a linear growth process so that specification uncertainty can be abrogated (see Fernadez, Ley and Steel; 2001, Brock and Durlauf; 2001, Sala-i-Martin, Doppelhoffer and Miller; 2004, Hendry and Krolzig; 2004, and Durlauf, Kourtellos, and Tan; 2007). However, an emerging theme in the literature (see Massoumi, Racine, and Stengos; 2007 for the most current research) has been the appearance of significant nonlinearities in cross-country growth regressions.[2] From this vista, it is relevant to identify the nonlinearities in the growth process so that they can be used to extend the model space of BMA and other growth model selection investigations in uncovering the appropriate growth process, assuming a universal one exists.[3]

Our ability to deal with specification uncertainty and variable uncertainty stems from recent research in nonparametric model selection methods, see Hall, Racine and Li (2004) and Hall, Li and Racine (forthcoming). These methods are robust to functional form misspecification (specification uncertainty) and have the ability to remove irrelevant variables that have been added by the research (variable uncertainty). We make a caveat that our variable uncertainty can be thought of as conditional; if a researched omits a relevant regressor from this exercise then the results provided may be erroneous and so we mention that removing the irrelevant variables is *conditional* on the variables included in the exercise. This is seen as an astounding quality of nonparametric methods as it provides yet another desirable characteristic that allows them to compete with standard parametric procedures. This type of approach is very important because Durlauf, Kourtellos and Tan (2007) argue that "the linear growth model (with constant parameters) may be misspecified," and further suggest that "more attention needs to be paid in

[1]See Brock and Durlauf (2001) and Durlauf (2003) for more on these four issues in the growth empirics literature.

[2]Kalaitzidakis, Mamuneas, and Stengos (2000) take a step in this direction by considering variable selection in the presence of possible nonlinearities, however, the main variables of interest enter into the model in a linear fashion.

[3]Masanjala and Papageorgiou (2007) have documented that African countries may grow differently than the rest of the world.

the literature to exploring possible nonlinearities and heterogeneities."

While the insights of the empirical growth papers employing BMA and model selection are valuable in and of themselves, their foundation of *a priori* functional form misspecification limits the scope of these methods in truly uncovering the process dictating economic growth. It may turn out that a variable found to be statistically relevant in explaining growth is arrived at through an inappropriate specification of the growth process. Here we feel that nonparametric model selection procedures are invaluable as a tool for uncovering the salient features of the growth process: those variables (conditionally) which are *relevant* for predicting growth and their *appropriate* influence on output.

Our results highlight the importance of accounting for nonlinearities across the spectrum of growth variables, including the Solow model variables themselves. We note that few specific growth theories outperform the baseline Solow specification with the exception of macroeconomic policy and institutions. These theories both have important policy implications for fostering long term economic growth and exploiting the intricate relationship that exist between the proxy variables used here and the growth of nations is an important prospect for future research.

The remainder of the paper is laid out as follows. Section 2 reviews the findings of various model selection studies to get a sense for the variables and theories that are most relevant for studying growth across countries. These results will serve as a benchmark and guideline for proper perspective of our results to follow. Section 3 provides the econometric intuition and the mechanics behind nonparametric model selection. Section 5 discuss the data to be used in the paper while Section 4 first provides Monte Carlo evidence that the nonparametric model selection methods work well for the sample sizes and number of included covariates typical of a growth regression exercise. The following section contrasts two and three growth theories at the same time across the 8 main theories listed in Durlauf, Kourtellos, and Tan (2007). Section 7 reviews the results and provides intuition for our findings. Our conclusions and suggestions for further research appear in Section 8.

2 Robustness and Specification - A Review

2.1 Growth Variable Robustness

Melding cross-country growth regressions with various conditioning sets dates back to the seminal work of Levine and Renelt (1992) who used Leamer's (1983) extreme bounds analysis (EBA) to check the robustness of the key economic, political and institutional variables that, at the time, were used extensively to detect empirical linkages with long-run growth rates. They looked at no more than seven growth variables at a time and focused on a cross section of anywhere from 64-106 countries depending on the variables used, investigating growth over the period 1960-1989. However, much of the study focuses on the shorter time horizon of 1974-1989 due to lack of specific conditioning variables for the period from 1960-1973. Levine and Renelt (1992) also adopted the tradition of including a set of variables that appear in 'every' regression run. Typically these are chosen to be the Solow variables[4], but are not required to be.

Their findings were novel, and to this day, still robust; only a handful of variables appeared to be able to withstand varying conditioning sets used for cross-country growth regressions. They tested variables from competing theories such as macroeconomic, trade, fiscal, and monetary policy as well as political stability of the country. The results suggested that initial income was robust while population growth was not. They also found that many of the proxy variables for the growth theories they were interested in were fragile. These variables ranged from political indicators to trade policy measures and on to fiscal policy. In sum, their analysis suggested that the key variables of the Solow model were the most robust of all the variables considered.

Their work was important on many levels, it represented a unification of competing growth variables in a tractable framework, it brought to the fore issues such as model uncertainty and theory openendedness,[5] and the role that both play on not only understanding the process of growth, but the impact that competing theories have on the conclusions that one draws from these types of exercises.

While the intuition and the conclusions laid out by Levine and Renelt (1992) have hardly been disputed, the means to which they arrived at those findings has. EBA can be seen as overtly restrictive in the face of non-robustness. To remedy this Sala-i-Martin (1997a,b) developed alternative methods that still penalized non-robust variables, albeit less harshly than EBA. Sala-i-Martin had 62 covariates but chose to follow the strategy of Levine and Renelt (1992) and only considered seven variables at a time and always included the Solow variables in every regression.[6] Sala-i-Martin (1997a,b) reached an almost polar conclusion than Levine and Renelt (1992) documenting 22 variables that were robust according to his method as well as all three of the Solow variables included in all the regressions. While his methods were not based on any formal statistical theory, they did open up a debate on the relevant sources of growth and how one goes about parsing them out from a seemingly infinite pool of candidate variables.

2.2 Model Uncertainty and Model Averaging

One point worth making is that the studies of Levine and Renelt (1992) and Sala-i-Martin (1997a,b) dealt with the robustness of key variables in cross-country growth regressions across varying specifications of said regressions. However, it was not until the turn of the century that growth empiricists started attacking the issues raised by Levine and Renelt (1992) and Sala-i-Martin (1996, 1997) with model averaging methods, acknowledging that the model space for cross-country growth regressions was quite large. To wit, Brock and Durlauf (2001) Fernandez, Ley and Steel (2001), Durlauf, Kourtellos, and Tan (2007) and Masanjala and Papageorgiou

[4]The traditional Solow variables are taken as initial income, population growth plus a constant designed to capture depreciation rates and technological growth, the investment rate, and a measure of human capital.

[5]It should be mentioned that these terms do not appear in Levine and Renelt (1992) but were alluded to. In fact these terms were really put forth in a growth context in Brock and Durlauf (2001).

[6]Sala-i-Martin (1997a,b) did not include population growth as one of his ‘Solow’ variables, thus he only has three variables that are in every regression.

(2007) have all attacked the robustness of various growth theories (for various countries) using BMA, while Sala-i-Martin, Doppelhoffer and Miller (2004) have used Bayesian averaging of classical estimates (BACE) procedures, while Hendry and Krolzig (2004) and Hoover and Perez (2004) used general to specific modelling approaches. These methods are more parsimonious that EBA and are grounded in statistical theory (see Hoeting, Madigan, Raftery and Volinsky; 1999 for a nice overview of BMA).

The findings from these various papers have varied conclusions. Fernandez, Ley and Steel (2001) use the same data as Sala-i-Martin (1997a,b) but do not require that only seven variables appear at a time and also do not include the Solow variables in every regression. Using a posterior probability cutoff of 90% they find that of the 22 variables deemed 'robust' in Sala-i-Martin (1997a,b), only four (initial income, percent Confucian, life expectancy and equipment investment) are statistically relevant from their perspective. Their findings were appealing for a variety of reasons, one of the most important being that the regressions considered were not required to have at most seven variables. This lent further evidence that limiting the size of the model space of linear growth regressions had an impact on the findings.

Brock and Durluaf (2001) laid the terminology and foundation for the importance of model averaging when considering growth models and growth theories. Their discussion of model uncertainty brought to light several key facets of model uncertainty: theory uncertainty, functional form uncertainty and heterogeneity uncertainty. Their application of BMA focused on the study of Easterly and Levine (1997) on the impact of ethnic conflict on growth and its potential for explaining Africa's dismal growth performance compared to the rest of the world. Brock and Durlauf (2001) find that ethnic conflict is a robust predictor of growth in the face of theory uncertainty. They also consider heterogeneity uncertainty by interacting all of the variables with a dummy for the Sub-Saharan countries of Africa. This was novel for two reasons. It was the first study to consider the potential for interactions within the Solow model/model uncertainty paradigm and it showed that ethnic conflict mattered exclusively for Sub-Saharan countries but not in the rest of the world. In fact coefficient estimates for this variable were from 7-10 times larger in Africa than in the rest of the world. Their study was limited in scope and focused almost exclusively on one particular variable, yet it was an enlightening point made within the confines of model uncertainty.

Building on the fact that Africa may grow differently than the rest of the world found in Brock and Durlauf (2001), Masanjala and Papageorgiou (2007) conducted a full scale study of model uncertainty focusing exclusively on Sub-Sahara African countries.[7] Their findings revealed that initial income, the fraction of mining in GDP and primary exports have posterior probabilities above 0.9 while the measure of ethnic conflict used by Brock and Durlauf (2001), ethnolinguistic fractionalization, had a posterior probability of only 0.39, suggesting that it was not a key determinant of growth in Africa when considering only African growth.

Durlauf, Kourtellos, and Tan (2007) represents the most comprehensive study of model uncer-

[7]They also considered a model including other countries from around to world as a baseline to show that Africa did indeed grow differently.

tainty of growth regressions to date. Their study considers an unbalance panel of countries and considers not only the importance of individual variables on the growth process, but the competing growth theories themselves. They also account for endogeneity in their model averaging exercises. Their findings suggest that many of the 'nouveau' growth theories are not as important as previously believed and that very few of the variables used have high posterior probabilities. The ones that rise to the top are initial income and investment (two of the Solow variables), government consumption and inflation (variables classified as relating to macroeconomic policy), and the East Asian regional dummy (related to regional heterogeneity). These results suggest that the growth theories that are most relevant for explaining growth are the original Solow model, macroeconomic policies and regional disparities across nations. Theories such as institutions, demography, geography, religion, and fractionalization do not appear to be robust theories of cross-country economic growth.

2.3 Nonlinearities and Heterogeneity in Growth Regressions

The robustness and model uncertainty exercises have shed new light on important and telling growth features. However, one area where these methods have been less used has been examining the impact of nonlinearities and parameter heterogeneity within the growth process. In fact, very few studies have paid much attention to the fact that growth may not be dictated by a global linear process. Durlauf and Johnson (1995) was the seminal empirical work that brought to the fore heterogeneity in cross-country growth. Their work showed that different countries obeyed different *linear* growth processes using regression tree methods. They were able to account for parameter heterogeneity within the standard Solow framework, albeit using a linear model.

Lee, Pesaran and Smith (1997) estimated a stochastic Solow model that allowed for parameter heterogeneity by considering a panel data setting. Their findings showed significant heterogeneity in terms of the speed of convergence, typically taken as a transform on the coefficient of initial income. These findings have subsequently been reaffirmed by Durlauf, Kourtellos, and Minkin (2001) and Kourtellos (2003) using semiparametric smooth coefficient models. These two approaches are interesting because they model nonlinearities and parameter heterogeneity in a simultaneous fashion.

An interesting extension of the Durlauf and Johnson (1995) paper is the constant elasticity of substitution (CES) production function setup of Papageorgiou and Masanjala (2004). They assumed that national outputs were governed by a CES technology as opposed to the Cobb-Douglas function used in Solow(1956). Papageorgiou and Masanjala (2004) allowed different groups of countries to have different growth paths as in Durlauf and Johnson (1995), but by modelling output with a CES production function they were able to account for nonlinearities and parameter heterogeneity in the growth process simultaneous. This ability to do two things at once was not able to be exploited by Durlauf and Johnson (1995) due to the linear nature of the Solow model's growth predictions.

Tan (2004) used GUIDE (general, unbiased interaction detection and estimation) to aid in identifying clustering of countries that obey a common growth model. This methodology is similar in spirit to that of Durlauf and Johnson (1995) but the methods employed by Tan (2004) look for interactions between covariates, thus introducing nonlinearities, and have the tendency to provide fewer regression splits. The evidence relayed in Tan (2004) show that institutional quality and ethnic fractionalization define convergence clubs. These results strengthen the implications of the Azariadis and Drazen (1992) model of threshold externalities for economic growth.

Much of the focus on nonlinearities in empirical growth regressions has been due to the pioneering work of Thanasis Stengos. Liu and Stengos (1999) consider a partly linear growth specification, Massoumi, Racine and Stengos (2005) consider a fully nonparametric growth structure, and Mamuneas, Savvides, and Stengos (2007) consider a semiparametric smooth coefficient model. All of these studies have shown significant nonlinearities for a variety of variables on cross-country economic growth. Mention few other papers.

To our knowledge the only paper that has combined robustness of economic variables in a growth regression context while allowing for nonlinearities has been Kalaitzidakis, Mamuneas and Stengos (2000). Their work used EBA, as in Levine and Renelt(1992), but allowed for nonlinearities by setting up the growth regression in a partly linear framework. They allowed the Solow variables to enter the growth regression in a linear fashion, consistent with the Solow model predictions, but the auxiliary variables used in Levin and Renelt (1992) were allowed to enter in a nonparametric fashion. KMS tested the linear specification of the auxiliary variables and then used these robust models to ascertain the significance of any variable in standard EBA fashion. Their findings confirmed that investment has a robust impact on growth, however, the omitted nonlinearities of Levine and Renelt (1992) showed that at least one variable from every major policy group was robust, contrary to their conclusions. In sum, KMS note that "... the use of a simple linear regression framework is inappropriate for assessment of the specification of cross-country growth models and for addressing the robustness properties of variables that enter these models."

3 Nonparametric Model Selection

Standard growth regressions take the following (linear) form:

$$g_i = \beta' w_i + \gamma' z_i + \varepsilon_i \tag{1}$$

where w_i is a vector composed of the 'Solow' variables, initial income, physical capital savings rate, human capital savings rate, and the joint depreciation term on both types of capital,[8] while z_i is a vector of unknown length that contains variables associated with several alternative growth theories. The exact variables within the z_i vector is what typically gives rise to model uncertainty; while there are many growth theories none refutes the others and so an exact specification of

[8]The common $n_i + g + \delta$ term that includes population growth rate, technology growth rate, and factor depreciation rates, respectively.

Equation (1) becomes increasingly difficult as more growth theories are constructed. Brock and Durlauf (2001) refer to this inability of growth theories to reject one another as 'openendedness'. Empiricists have used BMA to uncover just what variables matter in both the x_i and z_i vectors, but to date have yet to break free of the linear growth structure implicit in Equation (1).

Now, consider a general growth specification taking the unknown form:

$$g_i = m(x_i) + \varepsilon_i, \qquad i = 1, ..., N \tag{2}$$

where x_i is the union of w_i and z_i and g_i is the growth rate of country i. Further, m is the unknown smooth growth process. For the argument $x_i = [x_i^c, x_i^u, x_i^o]$ we make distinct reference to data type; x_i^c is a vector of continuous regressors (initial income, capital savings rate, percent Confucian), x_i^u is a vector of regressors that assume unordered discrete values (geographic regions, OECD membership), and x_i^o is a vector of regressors that assume ordered discrete values (time, number of conflicts, trade openness). An additive, mean zero error is captured through ε_i.

3.1 Nonparametric Regression

In this section we describe Li-Racine Generalized Kernel Estimation (see Li and Racine 2004 and Racine and Li 2004) of equation (2). Ignoring for the moment the fact that irrelevant regressors may have been included in Equation (2), we discuss its estimation using standard kernel techniques. To begin we model the unknown relationship through the conditional mean, i.e. $m(x_i) = E[g_i|x_i]$. This allows us to write the regression equation at a given point as

$$\hat{m}(x) = \frac{\sum\limits_{i=1}^{n} g_i K_h(x, x_i)}{\sum\limits_{i=1}^{n} K_h(x, x_i)}. \tag{3}$$

where

$$K_h = \prod_{s=1}^{q} h_s^{-1} l^c \left(\frac{x_{si}^c - x_{sj}^c}{h_s} \right) \prod_{s=1}^{r} l^u \left(x_{si}^u, x_{sj}^u, \widehat{\lambda^u}_s \right) \prod_{s=1}^{p} l^o \left(x_{si}^o, x_{sj}^o, \widehat{\lambda^o}_s \right). \tag{4}$$

K_h is the commonly used product kernel (see Pagan and Ullah 1999), where l^c is the standard normal kernel function with window width $h_s^c = h_s(N)$ associated with the s^{th} component of x^c. l^u is a variation of Aitchison and Aitken's (1976) kernel function which equals one if $x_{si}^u = x_{sj}^u$ and λ_s^u otherwise, and l^o is the Wang and Van Ryzin (1981) kernel function which equals one if $x_{si}^o = x_{sj}^o$ and $(\lambda_s^o)^{\left|x_{si}^o - x_{sj}^o\right|}$ otherwise. See Li and Racine (2004) and Racine and Li (2004) for further details. Nonparametric regression of this type is known as local constant least squares (LCLS).

Equation (3) can be written in matrix notation to display it in a more compact form. Let $\mathbf{i}$ denote an $n \times 1$ vector of ones and let $\mathcal{K}(x)$ denote the diagonal n matrix with j^{th} element $K_h(x, x_j)$. Also, denote by g the $n \times 1$ vector of growth rates across countries. Then, we can express our LCLS estimator as

$$\hat{m}(x) = \left(\mathbf{i}'\mathcal{K}(x)\mathbf{i}\right)^{-1} \mathbf{i}'\mathcal{K}(x)g. \tag{5}$$

Another popular method of nonparametric regression, known as local linear least squares (LLLS), begins by taking a first-order Taylor expansion [9] of (2) around x, yielding,

$$g_i \approx m(x) + (x_i^c - x^c)\beta(x^c) + \varepsilon_i \tag{6}$$

where x^c refers to the continuous variables within x, $\beta(x^c)$ is defined as the partial derivative of $m(x)$ with respect to x^c. The estimator of $\delta(x) \equiv (m(x_j), \beta(x^c))'$ is given by

$$\widehat{\delta}(x_j) = \left[\sum_i K_h(x, x_i)\begin{pmatrix} 1 \\ x_i^c - x_j^c \end{pmatrix}\left(1, \left(x_i^c - x_j^c\right)'\right)\right]^{-1} \sum_i K_h(x, x_i)\begin{pmatrix} 1 \\ x_i^c - x_j^c \end{pmatrix} g_i. \tag{7}$$

The returns to the categorical variables are obtained separately. For example, the coefficient on a dummy variable in the growth model (OECD) for example is calculated as the counterfactual change in OECD status of a particular country (switches from one to zero), *ceteris paribus*. Consequently, the returns to the categorical variables also vary across observations. This type of analysis is not common in parametric and semiparametric procedures. See Li and Racine (2004) and Racine and Li (2004) for further details.

Equation (7) can be written in vector-matrix form to reduce the notational burden. Let $\mathcal{X}$ be an $n \times (1 + q)$ matrix with j^{th} row being $\left(1, \left(x_j^c - x^c\right)'\right)$. Here q represents the number of continuous variables appearing in the unknown function. Our estimator takes the compact form

$$\hat{\delta}(x) = (\mathcal{X}'\mathcal{K}(x)\mathcal{X})^{-1}\mathcal{X}'\mathcal{K}(x)g \tag{8}$$

3.2 Cross-Validatory Bandwidth Selection

Estimation of the bandwidths $(h, \lambda^u, \lambda^o)$ is typically the most salient factor when performing nonparametric estimation. For example, choosing a very small h means that there may not be enough points for smoothing and thus we may get an undersmoothed estimate (low bias, high variance). On the other hand, choosing a very large h, we may include too many points and thus get an oversmoothed estimate (high bias, low variance). This trade-off is a well-known dilemma in applied nonparametric econometrics and thus we usually resort to automatic selection procedures to estimate the bandwidths. Although there exist many selection methods, Hall, Li, and Racine (2004, forthcoming) have shown that Least Squares Cross-Validation (LSCV) has the ability to smooth away irrelevant variables that may have been erroneously included into the unknown

[9] The Taylor expansion is only taken for the continuous variables.

regression function. Specifically, the bandwidths are chosen to minimize

$$CV(h, \lambda) = \underset{\{h,\lambda\}}{argmin} \frac{1}{n} \sum_{i=1}^{n} (g_i - \hat{m}_{-i}(x_i))^2, \tag{9}$$

where $\hat{m}_{-i}(x_i)$ is the common leave-one-out estimator. Notice that even when one is selecting bandwidths to be used for LLLS estimation, the unknown function is all that enters into the CV criterion, *not* the partial derivatives.

For the discrete variables, the bandwidths indicate which variables are relevant, as well as the extent of smoothing in the estimation. From the definitions for the ordered and unordered kernels, it follows that if the bandwidth for a particular unordered or ordered discrete variable equals zero, then the kernel reduces to an indicator function and no weight is given to observations for which $x_i^o \neq x_j^o$ or $x_i^u \neq x_j^u$. On the other hand, if the bandwidth for a particular unordered or ordered discrete variable reaches its upper bound, then equal weight is given to observations with $x_i^o = x_j^o$ and $x_i^o \neq x_j^o$. In this case, the variable is completely smoothed out (and thus does not impact the estimation results). For unordered discrete variables, the upper bound is given by $(d_s - 1)/d_s$ where d_s represents the number of unique values taken on by the variable. For example, a categorical variable for geographic location which takes on 5 values would have an upper bound for its bandwidth of $4/5 = 0.8$. For ordered discrete variables, the upper bound is unity. See Hall et al. (2004) for further details.

3.3 The Guts of Nonparametric Model Selection

The abundance of asymptotic results that form the statistical backbone of nonparametric methods have always assumed that the bandwidth(s) converge to zero (at a certain rate) as the sample size gets larger. This means that as the sample size is increased the amount of data in a specific region is growing and so the kernel weighting function no longer needs to use points farther away to construct an accurate representation of the functional form. However, recent advances have shown that when the researcher includes irrelevant variables, this bandwidth condition is no longer true. Automatic bandwidth selection procedures actually increase the bandwidths associated with irrelevant regressors, essentially removing them from the sample. It is as if the researcher had failed to include them in the first place! It was commonly believed that the inappropriate inclusion of irrelevant variables harmed the performance of nonparametric methods, but this is not the case.

In a set of papers, Hall, Li and Racine (2004, forthcoming) have shown that the inclusion of *irrelevant* regressors does not add to the 'curse of dimensionality'.[10] Their papers show that when one uses cross-validation procedures to select the appropriate amount of smoothness[11] of the unknown function, the covariates that are irrelevant are eliminated from the smoothing relationship. This property allows nonparametric estimators to not only allow for functional form misspecification, but relevant covariate selection at the same time. Thus tenets (ii) and (iii) al-

[10] Addition of other relevant variables still adds to the dimensionality issue however.
[11] See the Monte Carlo exercises in subsection 4.

luded to in the introduction can be handled simultaneously; a potentially elucidating advance for the growth empirics literature.

However, there is no free lunch for this method as it hinges on several facets that need to be considered on a case by case basis. First, the key assumption used by HLR (forthcoming) asks that the irrelevant regressors are independent of the relevant regressors, something unlikely to hold in practice.[12] Second, it is not entirely clear how well this method works as the set of relevant regressors is increased. HLR's finite sample investigations looked at at most two relevant regressors while there empirical application considered six variables for 561 observations in which only two regressors were deemed relevant according to their procedure. Clearly more work needs to be done to assess the performance of this level for very small sample sizes and for large sets of potential regressors.

4 Monte Carlo

Before formally analyzing any growth theory we feel it pertinent to assess the (very) small sample properties of nonparametric model selection in the face of more than one relevant covariate as well as many irrelevant covariates. This should lend credibility and insight into our assessment of growth theories found below. We notice that due to lack of information on certain variables that for any given theory we have samples as small as 167 countries and as large as 271. Therefore we conduct our small sample analysis using $n_1 = 100$ observations. Our setup follows Hall, Li and Racine (forthcoming), except that we include more relevant and irrelevant regressors. We judge the performance of the nonparametric model selection exercise based on out-of-sample predictive performance and the behavior of the cross-validated bandwidths.

To be firm, for $i = 1, \ldots, n$, with $n = 100$ we generate the following random variables: $(z_{1i}, z_{2i}, z_{3i}) \in \{0, 1\}$, $Pr[z_{1i} = 1] = .62$, $Pr[z_{2i} = 1] = .71$, $Pr[z_{3i} = 1] = .82$, $(w_{1i}, w_{2i}) = \{0, 1, \ldots, 3\}$, with $Pr[w_{1i} = \ell] = .25$, $\forall \ell$ and $Pr[w_{2i} = 0] = .4$ and $Pr[w_{2i} = \ell] = .2$, $\ell \in \{1, 2, 3\}$, while $(x_{1i}, x_{2i}, x_{3i}, x_{4i}, x_{5i})$ are all distributed normally with mean zero and variance one. The variables are drawn so that they exhibit a 0.50 degree of correlation.

We generate y_i according to

$$y_i = z_{1i} + x_{1i} + x_{2i} + x_{1i} \cdot x_{2i} + \varepsilon_i,$$

or

$$y_i = z_{1i} + \sqrt{w_{1i}} \cdot x_{1i} + x_{2i} + x_{1i} \cdot x_{2i} + x_{3i}^2 + \varepsilon_i.$$

For both models ε_i is drawn from a $\mathcal{N}(0, 1)$ distribution. In each model there is more than one relevant continuous variable and there are both categorical and continuous variables that are

[12] This is not entirely damning as it was shown in finite samples that the HLR method worked even when dependence was allowed between relevant and irrelevant regressors. The assumption was made for ease of proof of the corresponding theorems in the paper. Indeed, in our small sample exercises we violate this condition and it appears to have no affect on the corresponding results.

irrelevant. Both setups also contain nonlinearities to fully highlight the nonparametric approach. We feel that while limited these two models should provide good insight into how this method performs with a small sample and more than one relevant continuous covariate. Indeed, both Fernandez, Ley, and Steel (2001) and Sala-i-Martin, Doppelhofer, and Miller (2004) have both shown using BMA(BACE) that four continuous variables are a part of the true growth model with very high probability.[13]

Our first assessment is the ability of the cross-validation procedure to smooth away the variables that are indeed not present in the data generating process. We use LCLS to assess if both continuous and discrete variables have been correctly smoothed away. For the categorical variables we use the rule of thumb that if the bandwidth is within 5% of its upper bound that the variable has been smoothed out and for the continuous variables we look at the bandwidth compared to the standard deviation of the data drawn. If the bandwidth is larger then two standard deviations we conclude that the continuous variable has been smoothed out of the exercise. For our 1000 replications we note the median, 10th and 90th percentiles of the cross-validated bandwidths.[14] We see from Tables 1 and 2 that median results suggest that the method is correctly smoothing away irrelevant discrete and continuous variables. We note again that this is also for data that are drawn to have a 0.5 degree of correlation, lending further evidence that the method works well when variables are correlated.

Our second assessment involves the model's predictive performance where we generate data, *independent* from the original draw, from the same DGP of size $n_2 = 1,000$. Predictive performance is assessed via $PMSE = 1/n_2 \sum_{j=1}^{n_2} (\hat{y}_j - y_j)^2$. We consider three parametric models, an incorrect linear model (PI-ALL) that includes all the variables, an incorrect linear model that only includes the relevant variables (PI-ONLY) and the correct nonlinear, interactions model (PC) as well as the LCLS cross-validated results. Table 3 suggests that while the *correctly* specified model dominates all the competitors, the performance of the nonparametric model relative to the two incorrect models is notable. The relative performance is 37% better than the linear model with every variable included and 34% than the incorrectly specified model with only the relevant variables.

5 Data

Our data come from Durlauf, Kourtellos, and Tan (2007) (DKT hereafter) and represent an ample portion of the set of variables that have been used at one juncture or another to assess a growth theory.[15] We briefly look at several key features of the data before getting to our main results.

[13] The four that each found are different, with the exception of initial income, but they still both narrow down the large set of potential covariates to a relatively small set that is manageable for empirical studies employing nonparametric estimation methods.

[14] All bandwidths were found using the constrained optimization solver in Gauss 6.0.

[15] The theories tested and the variables used are contained in an appendix available from the authors upon request.

The DKT data set contains data for the traditional Solow model (initial income, investment rate, human capital, population growth) as well as variables that compose several of the contending growth theories being debated today: fractionalization, institutions, demographics, geography, religion, and macroeconomic policy. At least two variables for each theory are used. Given that region is an unordered discrete variable, which does not affect the asymptotic properties of the estimator, we include it when we compare all other theories.

6 Results

Given our small sample findings we feel the nonparametric model selection is applicable to the analysis of growth models given the small sample sizes commonly encountered. While we cannot place too much faith in our 'Kitchen Sink' investigation (167 observations with 24 continuous variables), the consideration of individual theories shed light into which variables are relevant, the potential for parameter heterogeneity and the existence of nonlinearities. These results alone are important and worthwhile. In fact, our results serve two distinct arenas within the empirical growth literature: discerning nonlinearities within any given theory and discerning the appropriate structure of parametric models to be used in model averaging and variable robustness exercises.

Our bandwidths for the exact DKT sample are presented in Tables 4 and 5. We present two sets of bandwidths because those found from the local constant setting are informative only of relevance for continuous variables, while those found in the local linear setting are informative only of linearity. For the two discrete variables, time and region, their relevance is determined in either method, although there is no reason the values should be equal. Again, for the continuous regressors, the upper bound of the bandwidth in the local-constant least-squares case determines whether or not the variable is relevant. The upper bound here is infinity and thus is impossible to observe in practice. We follow the suggestion of Hall, Li and Racine (2007) and use two standard deviations of the independent variable as the bound for relevance. Similarly, the estimated bandwidths for the continuous regressors determined by the local-linear least-squares estimator gives the relevance of the variable in terms of its 'linearity'. If any bandwidth exceeds two standard deviations of its associate variable, we conclude that it enters linearly. However, this linearity does not mean that important interactions do not exist. One should also check for these in practice.

To increase the efficacy of our model selection exercises we also divided up the DKT data to maximize the number of observations for any given theory. Thus, in Tables 5 and 7 we present bandwidths found for both local constant and local linear regressions with all available observations for a given theory.

6.1 Solow Model

Our bandwidths for the Solow variables, when considering *only* the Solow model, provide a snap shot of the model's perceived fit when viewed as the main driver behind economic output. We note that population growth and human capital are smoothed out while there appear to be relevant nonlinearities occurring in both investment and initial income. The nonlinearities in initial income are in accord with the findings of Durlauf, Kourtellos, and Minkin (2001) as well as Kourtellos (2003). Aside from a handful of studies, most growth researchers ignore any type of nonlinear structure either between or across these variables, often resorting to standard fare linear models.

It interesting to note that human capital is smoothed out in every setting except macroeconomic policy and institutions. Also, we note that just the inclusion of regional effects greatly improves the model's fit, bumping up the pseudo-R^2 from just under 0.5 to 0.73. This is similar to the results of Temple (1998) who found that there were significant regional impacts on output. We mention in passing that both investment and initial income are each relevant across all theories except that the institutions theory drives out the relevance of investment and the demography theory eliminates the relevance of initial income.

Moving to the local linear results we see that while initial income and investment are relevant across the space of theories, assessing their perceived linearity is a bit harried. In the demography, fractionalization, and institution theories, initial income enters in linearly, albeit relevantly, except in the demography theory. The linearity of investment is relevant across the geography and institution theories, with investment being irrelevant within the institutions theory to begin with. Thus we again confirm that in general both investment and initial income are relevant predictors of economic growth and display a nonlinear effect.

However, as noted in the previous section, the finite sample results for the nonparametric procedures improve drastically as more data is added. Table 6 shows the local-constant least-squares results for the larger data set. Several striking features are immediate. First, both investment and initial income are relevant across *all* theories individually. Second, population growth and human capital start to appear relevant across a wider array of theories than in the limited, homogenous sample. Lastly, the Solow model by itself fits the data much better than in the limited sample; a difference in fit of almost 0.17. Viewing Table 7 we see that initial income still appears to affect growth in a nonlinear manner, except in the geography, fractionalization, and religion theories. For investment, it always enters in the model in a linear fashion, a stark difference from our smaller sample results. In fact, it appears that the only Solow variable that is robustly nonlinear across the individual theories is initial income, something past research has touched upon.

We also see that region is never smoothed away across all theories, again suggestive of the research of Temple (1998). We note however that since the bandwidth on region is not zero that there exist important interactions between region and the continuous variables entering the model that are not captured in the Temple (1998) setting.

6.2 Individual Theories

While examining the impact of the Solow variables on economic growth is interesting and insightful, much of the focus on economic growth has focused on alternative explanations aside from factor accumulation and initial conditions. Theories such as geography and institutions have permeated the literature in recent years and created quite a stir among academics.[16] To determine how each theory *on its own* affects growth aside from factor accumulation, as well as the variables that may be seen as suitably characterizing the theory under consideration, we keep the same Solow variables, as well as region and time effects, in the models.

In terms of improvement in model fit we see from Table 4 that geography, macroeconomic policy and institutions are the highest among the individual theories for the homogeneous data set. While fit is only one way to judge the adequacy of a model we mention in passing that these three theories are the most intensely studied of those considered in this paper and in all three theories, more than one of the proxy variables are relevant and at least one of them enters in a nonlinear fashion. The fit of the macro model seems to slightly degrade when using local linear least squares while that of fractionalization improves by almost 10%, see Table 5. When looking at model fits for the heterogeneous samples, Tables 6 and 7 show that macro policy, geography and institutions dominate the other models when looking at the local constant regression while the fit of the geography model degrades somewhat in the local linear regression.

For the geography theory both the Koeppen-Geiger measure and % ice free coast measures are relevant and enter in nonlinearly. We see the same story emerge in the macro policy setting with all three of our proxies, openness, government consumption, and inflation all being relevant, however, only inflation appears to have a nonlinear impact on growth. Our setting for studying institutions uses four proxy variables, of which three are relevant and only expropriation risk entering in a relevant and nonlinear fashion.

This is suggestive that future research focusing exclusively on any of these individual theories should consider nonlinear impacts of the proxy variables. In fact, given that no variable completely captures the underlying theory being investigated, it is useful to have a means to discern both relevance and impact simultaneously, which is exactly what these nonparametric model selection techniques give us.

The demography theory set up provides a considerable improvement in fit over the basic Solow model, however, it is the only theory that suggests initial income is irrelevant. What's more, three of the four Solow variables are deemed irrelevant in the demography theory setup; the only theory of the six that displays this type of behavior. Of the demography variables, fertility and the reciprocal of life expectancy at the age of one, fertility is seen to be irrelevant while our life expectancy measure is relevant and enters in a nonlinear fashion. It is interesting to note from this theory that after region and time effects have been controlled for the implications of the model are to increase investment in both capital and health.

Our last two individual theories under consideration, fractionalization and religion are the worst fitting of the six theories, however, each predicts that three of the four Solow variables are relevant for explaining growth and the religion theory shows that these same three relevant predictors enter in a nonlinear manner. The fractionalization setup shows that the of the three relevant Solow variables, only investment enters in nonlinearly.

[16]See the papers by Rodrik, Subramanian, and Trebbi (2004) and Sachs (2003) for one glimpse of the ongoing debates over the causes of growth.

If we compare these results for the homogeneous data set to those of the larger heterogeneous data sets we reach some striking similarities. First, every theory has at least one proxy variable that is relevant. Second, at least one proxy variable from each theory enters the model in a nonlinear fashion. This is suggestive that there are numerous sources of economic growth and that nonlinearities play an important role in determining growth.

From the larger data set exercises we also note a few additional aspects afforded from the larger sample. In the demography theory, the relevance of the proxy variables has switched. Previously, life expectancy was a relevant predictor for growth, but it now appears that it is irrelevant and in fact fertility is driving predictions of growth. The Koeppen-Geiger measure remains relevant moving to a larger sample, while the percentage of land within 100km of ice free coast has turned irrelevant. Our macro theory proxy variables appear to be robust to the addition of almost 100 more observations with all three variables under study again showing relevance and as we see from Table 7, nonlinearly. Both fractionalization variables appear to impact growth in a relevant and nonlinear fashion while many of the religion variables are smoothed away. The additional six observations to the institutions theory have not shed new light on the variables being used as again two variables are relevant and impact growth nonlinearly.

7 Discussion of Findings

Our results for the singular theories follow along the lines of DKT, we see that several of the Solow variables, most notably initial income, are robust when switching across theories. In contrast however, we see that while initial income is a relevant regressor for explaining growth, its appearance in the growth model seems to suggest a nonlinear impact on growth rates. We also agree with DKT that the macro variables seem to generate the greatest improvement in fit over other theories and once again that these macro variables display a nonlinear effect on overall country growth. This is suggestive that both the BMA results of DKT and the nonparametric model selection techniques employed here are coming to the same conclusions about which variables impact growth, but are differing in the explicit nature of that impact.

8 Conclusion

This paper has offered a unique perspective into the debate over 'relevant' growth theories while allowing for specification uncertainty. The use of nonparametric modelling techniques allows the inclusion of irrelevant variables at no harm to the predictions of the model due to the ability to automatically remove them. This is an appealing feature of nonparametric methods in general and is critical for studying growth given recent findings that the growth process may be highly nonlinear.

Our findings have suggested that, across all theories, several of the Solow variables are relevant predictors of economic growth and do so in a nonlinear fashion. We also found that investment and initial income appear to be the most robust Solow predictors over all theories. Aside from our conclusions regarding the Solow variables, we found that macro economic policy, geography, and institutions appeared to be the individual front runners when looking at theories individually. Also, each of these theories had numerous relevant proxy variables and nonlinearities present, suggesting more attention is necessary in the discussion of which of these theories is best.

Our findings are only preliminary and suggest that future research should take notice of the potential for nonlinearities and interactions across variables to uncover the full impact of any growth theory. As more data becomes available both for variables within a theory as well as for countries in general, these model selection methods will prove invaluable, given the results from our small sample exercises. Indeed, even looking at the individual theories with an expanded, albeit heterogeneous, data set, we found that the all of the Solow variables began to display themselves as relevant across theories. We also reaffirmed many of the same insights drawn from the larger dataset, providing further credence to the small sample performance of the nonparametric model selection techniques.

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Table 1: Summary of cross-validated bandwidths for the discrete covariates NP LSCV estimator.

	Median, [10th Percentile, 90th Percentile] of $\hat{\lambda}$				
	$\hat{\lambda}_{z_1}$	$\hat{\lambda}_{z_2}$	$\hat{\lambda}_{z_3}$	$\hat{\lambda}_{w_1}$	$\hat{\lambda}_{w_2}$
Model 1	0.17	0.5	0.5	0.39	0.85
	[0,0.49]	[0.22,0.5]	[0.23,0.5]	[0.25, 1]	[0.56, 1]
Model 2	0.24	0.50	0.50	0.41	0.85
	[0,0.5]	[0.02,0.5]	[0.02,0.5]	[0.01, 0.99]	[0.09, 1]

Table 2: Summary of cross-validated bandwidths for the continuous covariates NP LSCV estimator.

	Median, [10th Percentile, 90th Percentile] of $\hat{h}$				
	$\hat{h}_{x_1}$	$\hat{h}_{x_2}$	$\hat{h}_{x_3}$	$\hat{h}_{x_4}$	$\hat{h}_{x_5}$
Model 1	0.38	0.31	45.03	47.27	12.44
	[0.30, 0.53]	[0.19, 0.56]	[1.21, 53.74]	[20.84, 54.10]	[0.90, 50.16]
Model 2	0.40	0.51	0.41	26.74	31.60
	[0.16, 0.67]	[0.19, 3.50]	[0.15, 1.00]	[0.77, 4774]	[1.10, 4650]

Table 3: Out-of-sample predictive PMSE performance for parametric and nonparametric models containing irrelevant regressors for $n_1 = 100$ ($\rho = 0.5$).

	Median, [10th Percentile, 90th Percentile] of PMSE			
	NP-LSCV	PI-ALL	PI-ONLY	PC
Model 1	1.85	2.54	2.47	1.06
	[1.51,2.07]	[2.38,3.06]	[2.32,2.99]	[1.00, 1.12]
Model 2	3.11	8.38	8.15	1.12
	[2.11,4.59]	[7.18,9.24]	[7.19,9.26]	[0.97, 1.25]

Table 4: Bandwidths for DKT data using Local Constant Regression

Variable	Solow	Region	Demo	Geo	Macro	Frac	Rel	Inst	Kitchen Sink
Population Growth	410210	0.0802	160790	2800122	0.1422	0.1725	0.3510	0.0024	0.0049
Investment	0.1184	1.1490	0.6598	0.4721	0.2307	0.3958	0.4883	384712	0.0097
Human Capital	2282240	8580899	495056	7146979	0.4076	2545186	2.0254	0.1235	6121150
Initial Income	0.6716	0.3729	417718	0.3120	0.1472	0.2740	0.2563	0.0099	0.0412
Time	0.6755	0.7817	0.8355	0.4057	0.4194	0.9577	0.5581	0.1967	0.3282
Region	.	0.1076	0.3359	0.4285	0.0131	0.1499	0.3227	0.0000	0.8333
Fertility	.	.	783861	.	.	.	.	.	0.1287
Life Expectancy	.	.	0.0015	.	.	.	.	.	0.0001
Koeppen-Geiger	.	.	.	0.0688	.	.	.	.	1634105
% Ice Free Coast	.	.	.	0.2779	.	.	.	.	0.1736
Openness	.	.	.	.	0.2268	.	.	.	0.1838
Net Govt. Cons	.	.	.	.	0.0207	.	.	.	0.0367
Inflation	.	.	.	.	0.0700	.	.	.	0.0594
Language	.	.	.	.	.	0.1291	.	.	100359
Ethnic Tension	.	.	.	.	.	1992069	.	.	0.0722
Hindu	.	.	.	.	.	.	0.0586	.	7812
Jewish	.	.	.	.	.	.	3455412	.	192129
Muslim	.	.	.	.	.	.	1727793	.	3052186
Orthodox	.	.	.	.	.	.	40294432	.	0.1100
Other Religion	.	.	.	.	.	.	0.0332	.	0.0673
Protestant	.	.	.	.	.	.	533780	.	0.0713
Eastern Religions	.	.	.	.	.	.	0.0060	.	576543
Exec. Constraints	.	.	.	.	.	.	.	0.0250	19612
Exprop. Risk	.	.	.	.	.	.	.	0.1026	308674
KKZ96	.	.	.	.	.	.	.	1205089	0.1759
Legal Formalism	.	.	.	.	.	.	.	0.0648	0.0087
Model Fit	49.23	73.21	89.73	91.24	99.54	84.12	87.51	99.99	99.99

Table 5: Bandwidths for DKT data using Local Linear Regression

Variable	Solow	Region	Demo	Geo	Macro	Frac	Rel	Inst	Kitchen Sink
Population Growth	363128	0.1361	1057716	314653	0.2047	1.4589	0.1162	183568	0.1764
Investment	1.8282	2.3527	0.6598	1528366	0.6376	0.9153	0.5621	1247654	27352
Human Capital	1.3544	2920565	642330	3596212	774069	10579156	0.6186	245344	341092
Initial Income	1.0519	2.1663	1765849	0.6561	1.0178	954223	0.4876	672167	8.8872
Time	1	1	0.8355	0.8177	0.7607	0.6708	0.4915	1	0.7393
Region	.	0.1627	0.3359	0.3647	0.3062	0.4418	0.0493	0.3352	0.8333
Fertility	.	.	887142	.	.	.	.	.	26318
Life Expectancy	.	.	0.0015	.	.	.	.	.	7860
Koeppen-Geiger	.	.	.	0.1589	.	.	.	.	0.4348
% Ice Free Coast	.	.	.	0.3645	.	.	.	.	30439
Openness	.	.	.	.	224686	.	.	.	555605
Net Govt. Cons	.	.	.	.	75246	.	.	.	499721
Inflation	.	.	.	.	1.0149	.	.	.	88597
Language	.	.	.	.	.	0.2260	.	.	2293828
Ethnic Tension	.	.	.	.	.	0.0990	.	.	235696
Hindu	.	.	.	.	.	.	0.0271	.	18268
Jewish	.	.	.	.	.	.	0.0822	.	263049
Muslim	.	.	.	.	.	.	0.1939	.	677634
Orthodox	.	.	.	.	.	.	0.0082	.	0.3581
Other Religion	.	.	.	.	.	.	0.0506	.	290581
Protestant	.	.	.	.	.	.	0.0537	.	573461
Eastern Religions	.	.	.	.	.	.	0.1001	.	69656
Exec. Constraints	.	.	.	.	.	.	.	34051	7421
Exprop. Risk	.	.	.	.	.	.	.	0.2364	0.2358
KKZ96	.	.	.	.	.	.	.	0.5345	1311077
Legal Formalism	.	.	.	.	.	.	.	1510442	0.1839
Model Fit	48.05	71.69	82.82	92.43	88.37	92.05	90.67	99.90	99.99

Table 6: Bandwidths for Full Theory data sets using Local Constant Regression

Variable	Solow	Region	Demo	Geo	Macro	Frac	Rel	Inst
Population Growth	0.1427	0.1817	357571	0.4266	0.2139	0.3708	795171	1506792
Investment	0.1888	0.4698	0.5312	0.4576	0.4755	0.5192	0.3834	0.4570
Human Capital	1.1703	2.3535	2139032	0.7527	0.5129	0.9569	2974283	7445294
Initial Income	0.3945	0.4879	0.3877	0.5342	0.2711	0.5170	0.5367	0.2607
Time	0.5379	0.2110	0.5595	0.4586	0.1671	0.7268	0.5692	0.8634
Region	.	0.6389	0.1074	0.2216	0.5375	0.1188	0.5624	0.2247
Fertility	.	.	0.1769	.	.	.	.	.
Life Expectancy	.	.	9841	.	.	.	.	.
Koeppen-Geiger	.	.	.	0.1018	.	.	.	.
% Ice Free Coast	.	.	.	1461365	.	.	.	.
Openness	.	.	.	.	0.1625	.	.	.
Net Govt. Cons	.	.	.	.	0.0382	.	.	.
Inflation	.	.	.	.	0.1692	.	.	.
Language	.	.	.	.	.	0.1796	.	.
Ethnic Tension	.	.	.	.	.	0.8328	.	.
Hindu	.	.	.	.	.	.	0.1886	.
Jewish	.	.	.	.	.	.	272966	.
Muslim	.	.	.	.	.	.	8868439	.
Orthodox	.	.	.	.	.	.	0.0407	.
Other Religion	.	.	.	.	.	.	1778613	.
Protestant	.	.	.	.	.	.	1621897	.
Eastern Religions	.	.	.	.	.	.	0.0416	.
Exec. Constraints	.	.	.	.	.	.	.	0.2481
Exprop. Risk	.	.	.	.	.	.	.	8795918
KKZ96	.	.	.	.	.	.	.	1.2687
Legal Formalism	.	.	.	.	.	.	.	0.1206
Sample Size	271	271	267	256	265	247	269	173
Model Fit	66.03	61.48	69.60	81.14	97.62	72.96	63.42	92.07

Table 7: Bandwidths for Full Theory data sets using Local Linear Regression

Variable	Solow	Region	Demo	Geo	Macro	Frac	Rel	Inst
Population Growth	325616	734717	0.1768	1.1537	0.2865	388851	0.4981	227913
Investment	2.1206	8054010	3237715	4029275	1.5667	2.5226	2.6697	1060182
Human Capital	1.5165	2.1397	7581277	1.7506	3271349	1750581	1.7987	1016826
Initial Income	0.9804	0.9934	0.6631	3977940	0.4237	401174	5.7774	0.4815
Time	0.8860	0.4847	0.5322	0.6052	0.5590	0.5434	0.6435	1
Region	.	0.5518	0.4653	0.5130	0.6864	0.5810	0.6686	0.3727
Fertility	.	.	0.8755	.	.	.	.	.
Life Expectancy	.	.	0.0038	.	.	.	.	.
Koeppen-Geiger	.	.	.	0.2086	.	.	.	.
% Ice Free Coast	.	.	.	337765	.	.	.	.
Openness	.	.	.	.	0.7604	.	.	.
Net Govt. Cons	.	.	.	.	0.1786	.	.	.
Inflation	.	.	.	.	0.8172	.	.	.
Language	.	.	.	.	.	0.9043	.	.
Ethnic Tension	.	.	.	.	.	0.2206	.	.
Hindu	.	.	.	.	.	.	4.7708	.
Jewish	.	.	.	.	.	.	0.0994	.
Muslim	.	.	.	.	.	.	2.5299	.
Orthodox	.	.	.	.	.	.	0.2593	.
Other Religion	.	.	.	.	.	.	0.0831	.
Protestant	.	.	.	.	.	.	0.7395	.
Eastern Religions	.	.	.	.	.	.	0.1906	.
Exec. Constraints	.	.	.	.	.	.	.	0.5429
Exprop. Risk	.	.	.	.	.	.	.	1049243
KKZ96	.	.	.	.	.	.	.	2104962
Legal Formalism	.	.	.	.	.	.	.	0.2720
Sample Size	271	271	267	256	265	247	269	173
Model Fit	52.80	57.95	73.64	70.62	81.53	68.06	77.74	91.61