Vienna University of Economics & B.A.

Department of Economics Working Paper Series

Footloose capital and productive public services[*]

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Working Paper No. 111, October 2007

Abstract — We analyse in a Footloose Capital productive public services provided by a central government aiming at reducing regional disparities. Two countervailing effects occur – one upon productivity and another upon local demand – the relative strength of which depends upon the financing scheme. Only if the "rich" region contributes sufficiently to the financing of the public services in the "poor" region, the poor region will actually gain. In studying these questions we pay particular attention to the dynamic adjustment processes and to the role of trade freeness.

Keywords: New Economic Geography, public services, regional policy

JEL-Classification: H4, R1, R12, F12

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[*] We presented a companion paper in seminars held at the University of Naples (Italy, April 2007) and at the University of Insubria (Varese, Italy, June 2007), in Workshops in Lucca (PRIN-2005, Lucca, July 2007) and Kiel (The Kiel Institute for the World Economy, Germany, April 2007) and at the RIEF 2007 Conference (Rome, University of Tor Vergata, May 2007).

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By Pasquale Commedatore Ingrid Kubin and Carmelo Petraglia

1. Introduction

Brakman et al. (2005) claim that the European Cohesion Policy is incoherent since it seems sometimes targeting agglomeration of industrial activities in core regions, but more often stimulating their relocation in peripheral ones. Such a criticism provides a motivation to analyse policy issues in NEG models, whose main focus is on the determinants of spatial location of manufacturing industry [1]. However, the incorporation of public expenditures in New Economic Geography (NEG) models represents a very recent theoretical advance. Providing the full picture of the working of both agglomeration and dispersion forces induced by the policy measures aimed to make backward regions more attractive for investors might be helpful for the design of effective policies aimed at promoting a sustained catching-up process across UE regions.

The impact of public expenditures on the location decisions of firms has been studied within a few variants of NEG models. [2] In particular, in a companion paper (Commendatore et al., 2007), we consider a two-regions Constructed Capital (CC) model, i.e. a Footlose Capital (FC) model with the additional feature of creation and destruction/depreciation of capital goods. In that paper our main focus is on industrial location and welfare effects of productive public services provision under the assumption of endogenous capital. The

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[1] As the authors point out, on the one hand, billions of euros are devoted to the financing of infrastructure projects in core regions; on the other hand, the bulk of the available funds are directed towards lagging regions in the attempt to achieve a "balanced spread of economic activity" (Brakman et al., 2005).

[2] See, in particular, the contributions of Martin and Rogers (1995), Trionfetti (1997), Brülhart and Trionfetti (2004), Brakman et al. (2007) and Commendatore et al. (2007). For an overview on the main results in the literature the reader is referred to Commendatore et al. (2007).

government uses tax revenues to purchase capital goods to use in the production of freely available public services. Hence, public policy can affect production in the manufacturing sector via its impact on factors productivity. We show in that paper that the interplay of two effects determines the final impact of an increase in productive public spending in the backward region on the spatial distribution of firms: the productivity effect and the demand effect. On the one hand, an increase in the provision of public services in one region lowers labour input requirement and lead firms to relocate there (productivity effect); on the other hand it could favor dispersion via a change in the relative market size (demand effect). As a result, whether or not higher provision of public services leads firms to relocate in the backward region will depend upon the financing scheme of public expenditure. It is shown that the demand effect is negligible if the government lets the richer region's tax payers contribute on the basis of their contribution capacity.

In this paper we aim at providing further insights on how the interplay of the above mentioned productivity and demand effects influences industrial location. In order to do so, we consider a much simpler analytical framework, which allows fully characterizing the dynamic behavior of capital movements in response to variations in the degree of trade freeness and in the provision of public services.

We deal with a FC model without an investment sector. In the standard Core-Periphery (CP) model (Krugman, 1991) it is assumed that mobile workers spend their incomes locally and imperfectly competitive firms tend to locate in the biggest market, enforcing the so-called "home market" effect which leads to agglomeration. The "cost of living" effect – goods are cheaper in regions with higher concentration of industrial firms – also favors agglomeration. On the other hand, the market-crowding or competition effect – the presence of an increasing number of competing firms lowers retail sales and wages – favors dispersion. The standard result of the model is that, at a sufficiently low level of trade costs – the so-called "break point" –, when a migration shock – driven by real wage differentials – occurs, agglomeration forces overpower dispersion ones in such a way to end up with the concentration of all industry in one region. As a result, complete agglomeration in one region is a stable equilibrium.

The FC model departs from the CP model for two crucial assumptions: a fixed capital requirement for each variety of the differentiated good and workers international immobility. In contrast with Krugman (1991), the mobile factor (capital) earnings are repatriated and spent where the capital owner reside. Therefore the typical CP feature of demand linked circular causality – production changes brought about by factor movements yield expenditure switching that in turn generate further production changes – does not arise. Furthermore, since costs-of-living are irrelevant to the production location decisions of capitalists, the cost-linked circular causality of the CP model – shifts in production alter prices inducing workers migration with further production shifting – is eliminated. Hence, the CP outcome of catastrophic agglomeration in one region is ruled out; however, agglomeration still occurs due the working of the home market effect.

As policy is concerned, we assume that the government decides on the levels of the public services in both regions and on the contribution of the two regions to its financing (which may not be proportional to the amount of the public services provided to the region). We assume that public services are produced by using the agricultural commodity and increase the productivity in the local manufacturing sector, not having any impact on private utility.

Extending Commendatore et al. (2007), we pay attention to the dynamic structure of the model. First, we fully characterize the dynamic process underlying capital movements and analyze the long-run equilibrium given as fixed point of the capital mobility dynamics for different degrees of trade freeness. Second, we study the impact of the provision of public services on the long-term behavior of the regional shares of capital.

The paper is structured as follows. Section 2 provides empirical background on recent trends of public investment and critical factors influencing foreign direct investment (FDI) in European regions. In section 3 we introduce the assumption of our model; section 4 presents the full model short-run structure, explicitly specifying the capital migration process. Section 5 deals with the dynamics of the model, studying the stability properties of core-periphery and interior equilibria, while section 6 presents the decomposition of the final impact of the provision of public services on the long run allocation of capital across regions into the productivity and the demand effects. We run such a policy experiment under different degrees of economic integration. Finally, section 7 concludes.

2. Empirical Background

In order to enhance the attractiveness of backward regions, European regional policy heavily relies on public investment, viewed as "contributing directly to economic growth and strengthening the productive potential of the economy". Public investment includes investment in human capital and infrastructure as well as expenditure on education and training aimed at improving the skills of the work force.

As documented by the European Commission (2007), in recent years public investment in EU countries has experienced a declining path at the national level.[3] The share of public investment to GDP in the EU15 declined from 2.9% in 1993 to 2.4% in 2005. On the other hand, public expenditure at regional and local levels has been increasing annually by 3.6% between 2000 and 2005, faster than total public expenditure (2.4%). The main factors explaining this declining pattern are "the general tendency towards a shrinking public sector, the increased involvement of the private sector in public sector capital projects and the pressure to reduce overall public expenditure to comply with rules on the budget deficit".

Conversely, the management of public investment has gradually being decentralised to regional and local levels. As a result, the share of regional and local authorities in public investment increased in average in all UE countries, underlying the higher responsibility of local government in the allocation of development-related public spending.

In the view of the European policy maker, FDI flows are interpreted as a critical factor for the development of lagging regions, particularly for the new member States, representing the "primary way in which the productivity gap between the industries and services located there and those in the rest of EU can be narrowed" (European Commission, 2007, pag. 73). Policies are hence targeted to enhance the attractiveness of regions via the improvement of basic and ICT infrastructures and the education of the work force. However, what we refer to as the demand effect induced by public spending – public expenditure changes imply

[3] These figures are referred to "gross fixed capital formation" (dwellings, other buildings and structures, machinery and equipment, and computer software).

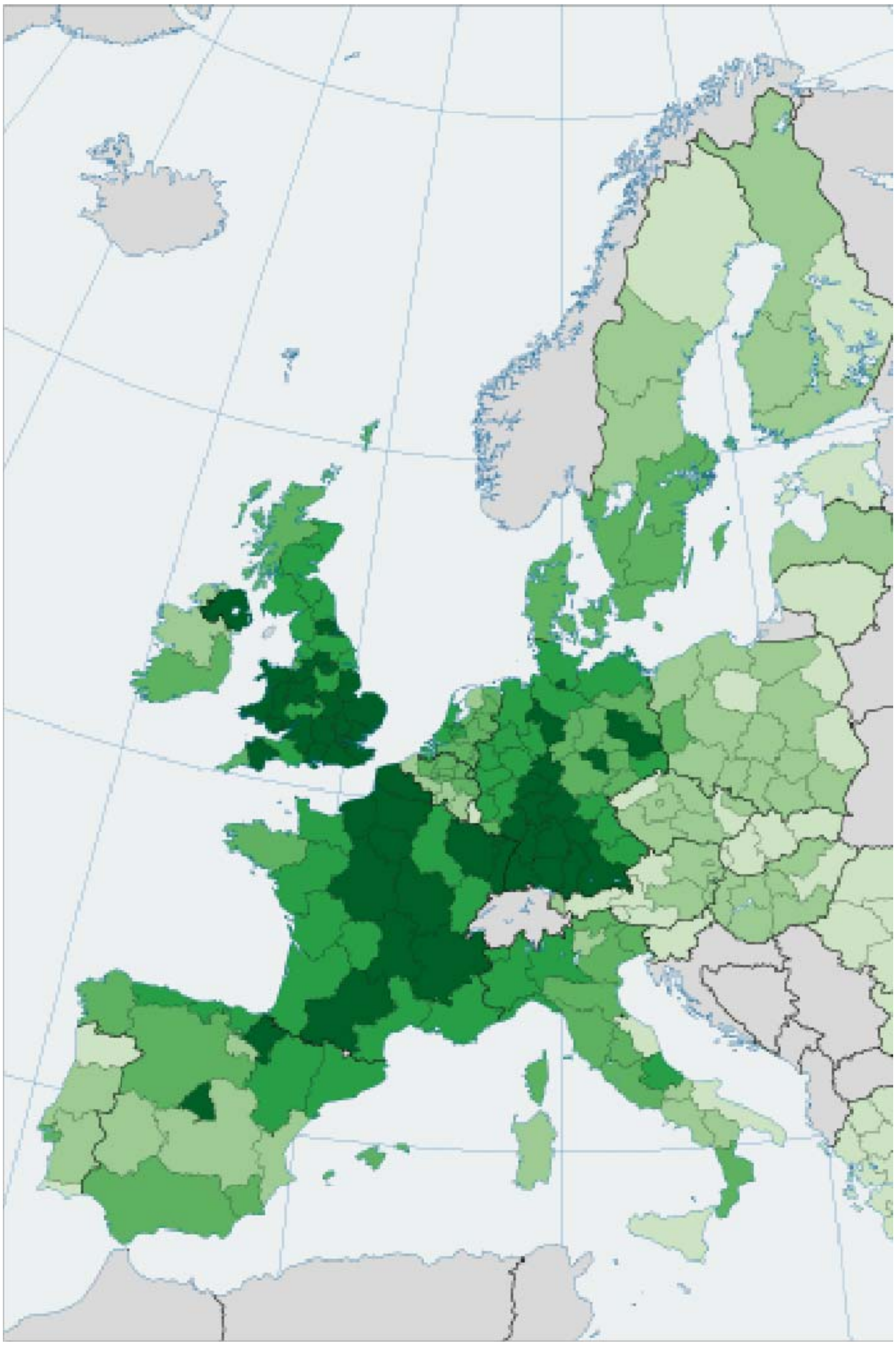
changes in regional market size – seems to be neglected by the policy maker according to whom “policy cannot affect factors such as national market size” (European Commission, 2007, p. 74). The neglecting of such an issue provides further motivation to our study.

Data on FDI account for a heterogeneous set of transactions, only part of which relating to actual location decisions. However, the scarcity of available alternative information makes them a natural proxy for capital movements across borders (Buettner, 2002).

FDI intensity – measured in terms of employees in foreign owned firms in relation to the total number of employees – across European regions is shown in figure 1. Regions with the largest shares are concentrated in the UK, Germany and France, while Spain only has two regions with a large share (Madrid and Navarra). The regions bordering France and the Atlantic also tend to have larger than average shares. The highest concentration is reached in Randstad regions, in Belgium, in Brussels and most of the Flemish regions and in Ireland (in the regions in which Dublin and Cork are situated). In contrast, all the new Member States, Finland, Greece, Portugal and southern Italy all have below average shares.

Figures 2-5 report estimated relative importance of factors that can potentially make backward European regions more attractive for foreign investors (Copenhagen Economics, 2006). Figures refer to four subsets of regions: “Eastern Europe”, “cohesion countries”, “regions facing weaknesses in competitiveness and employment” and “remote regions”. In each case, figures are constructed for the most attractive regions within the respective subset. The common pattern is that even if most attractive regions in each subset seem to be appealing for foreign investors, they still suffer from strong weaknesses with respect to the EU27 average, especially with respect to infrastructure and human capital.

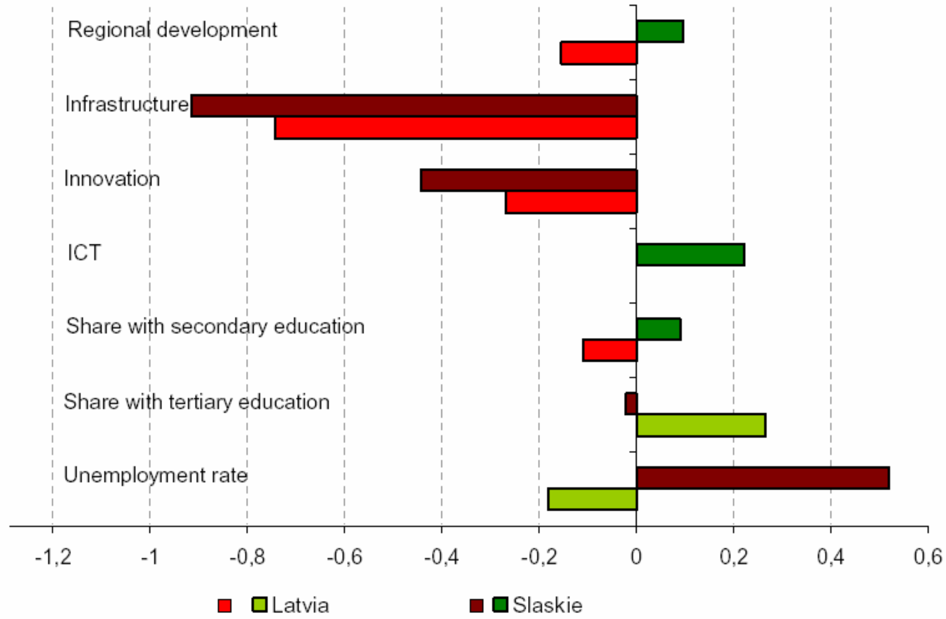
Figure 1: FDI intensity in 2004 (Employees in foreign firms as % of total number of employees)



Note: The darker the areas, the higher the intensity if FDI

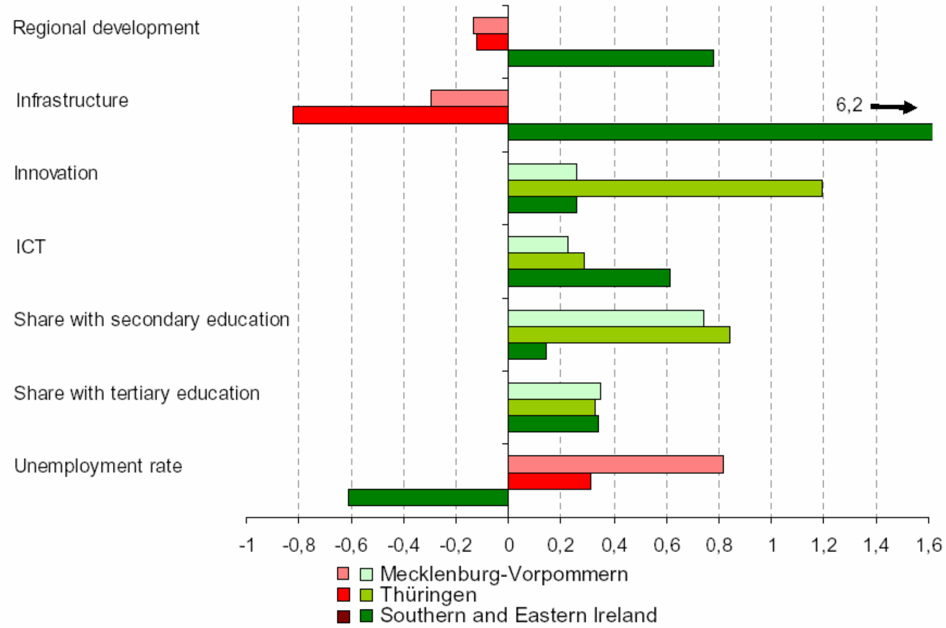
Source: Copenhagen Economics (2006)

Figure 2: Regional attraction factors of Latvia and Slaskie relative to the regional average in Eastern Europe



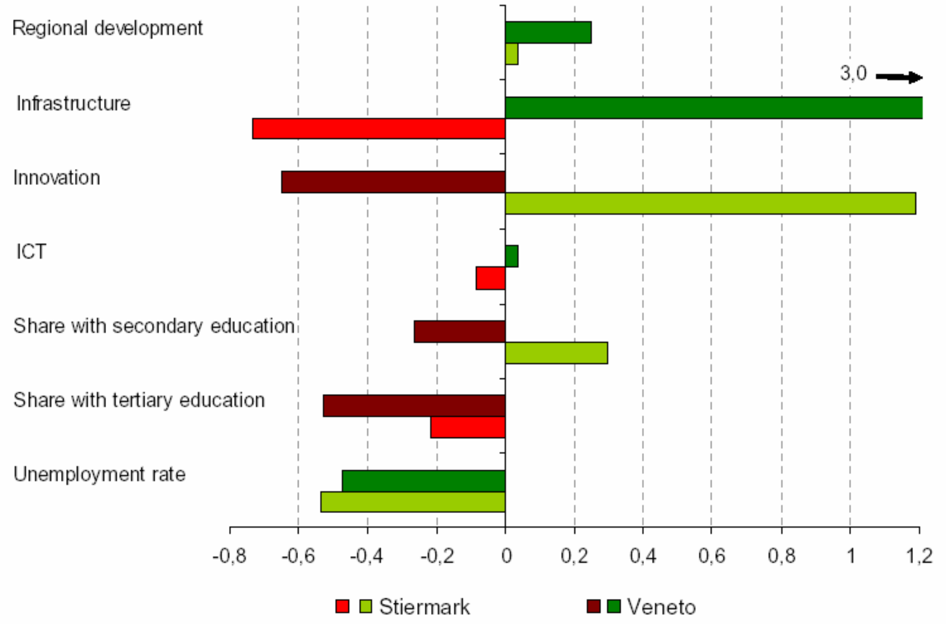
Note: The regional attraction factors are reported as the deviation from the regional average in Eastern Europe divided by the regional average in Eastern Europe. Green colours represent a better situation than the EU27 average whereas a less attractive situation is reported in red colours. ICT data for Latvia are not available.
Source: Copenhagen Economics (2006)

Figure 3: Regional attraction factors of Mecklenburg-Vorpommern, Thüringen and Southern and Eastern Ireland relative to the average of regions in the cohesion countries



Note: The regional attraction factors are reported as the deviation from the regional average of the cohesion countries divided by the regional average in the cohesion countries. Green colours represent a better situation than the EU27 average whereas a less attractive situation is reported in red colours.
Source: Copenhagen Economics (2006)

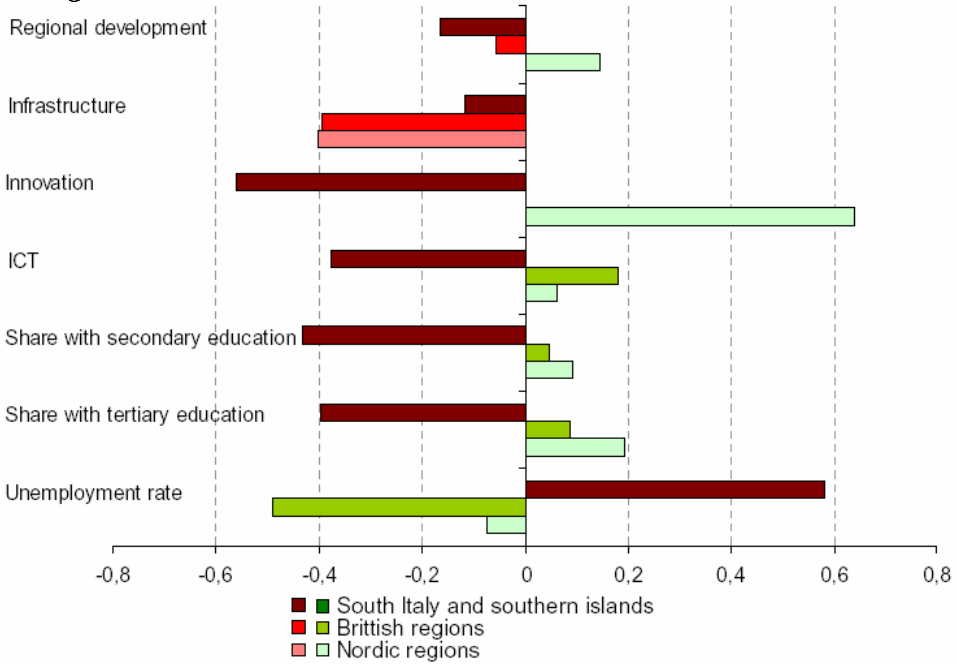
Figure 4: Regional attraction factors of Veneto and Steiermark relative to the average of regions facing weaknesses in competitiveness and employment



Note: The regional attraction factors are reported as the deviation from the regional average in the Cohesion countries divided by the regional average in the Cohesion countries. Green colours represent a better situation than the EU27 average whereas a less attractive situation is reported in red colours.

Source: Copenhagen Economics (2006)

Figure 5: Regional attraction factors of three groups of remote regions relative to the EU27 average



Note: The regional attraction factors are reported as the deviation from the EU27 regional average divided by the EU27 regional average. Green colours represent a better situation than the EU27 average whereas a less attractive situation is reported in red colours. Innovation data for the remote British regions are not available.

Source: Copenhagen Economics (2006)

3. Assumptions

The Footloose Capital model involves two countries or regions, $r = 1, 2$, each with a monopolistically competitive manufacturing sector and a perfectly competitive agricultural sector. There are, in total, L consumers with a share s_L located in region 1. Each consumer provides one unit of labour per period. Labour is immobile between regions but instantaneously mobile between sectors. Consumers are also the capital owners.

A key feature of the Footloose Capital model is that physical capital is mobile between regions but capital owners are completely immobile and they spend all their earnings in the region in which they live. Consequently, our notation must differentiate between regional shares in capital ownership: s_K is the share of physical capital *owned* by capitalists resident in region 1 and λ_t is the share of physical capital *located* and used in region 1 in period *t*.

A representative consumer has the following utility function:

(1) $$U = \left(C_A\right)^{1-\mu} \left(C_M\right)^{\mu} .$$

The private utility component depends in the usual form on quantity consumed of a homogeneous agricultural good, C_A, and on a quantity index C_M that is a CES function of the varieties of manufactured goods. The constant elasticity of substitution between the manufactured varieties is denoted by $\sigma > 1$; the lower σ, the greater the consumers' taste for variety. The exponents of the agricultural good and of the manufacturing composite in the common utility function – $\left(1-\mu\right)$ and μ, respectively – indicate the invariant shares of disposable income devoted to the agricultural good and to manufactures; therefore $0 < \mu < 1$.

We have one government that provides public services for the two regions H_r, which increase productivity in the regional manufacturing sector. For one unit of the public service one unit of the agricultural commodity is used; the respective production function is:

(2) $$H = C_{AG} .$$

Public expenditures are financed by income taxes, the tax rates may differ between region, the overall budget is always balanced.

The agricultural commodity is produced with labour as the sole input, one unit of labour yields one unit of the agricultural product. We assume that neither region has enough labour to satisfy the total demand of both regions for the agricultural good. Thus, both regions always produce the agricultural commodity – the so-called non-full-specialization condition. Transportation of the agricultural product between regions is costless.

Manufacturing involves increasing returns: each manufacturer requires a fixed input of 1 unit of capital to operate and has a marginal labour requirement $\beta_r = f\left(H_r\right)$ that depends (negatively) upon the locally provided public services, where $f' < 0$ and $f'' > 0$. A very simple function which satisfies these properties is

$$f\left(H_r\right) = A\left(1 + BH_r\right)^{-1},$$

where *A* and *B* are positive constants.

Transport costs for manufactures take an iceberg form: if 1 unit is shipped between the regions, $1/T$ arrives where $T \geq 1$. Following Baldwin et al. (2003), to compact the notation, we introduce the parameter $\phi \equiv T^{1-\sigma}$ which is conventionally labelled 'Trade Freeness', where $0 < \phi \leq 1$, with $\phi = 1$ corresponding to no trade cost ($T = 1$) and with $\phi \to 0$ corresponding to trade cost becoming prohibitive ($T \to \infty$). The manufacturing sectors involve Dixit-Stiglitz monopolistic competition. Given the consumers' preference for variety, a firm would always produce a variety different from the varieties produced by other firms. Thus the number of varieties is always the same as the number of firms. Furthermore, since 1 unit of capital is required for each manufacturing firm, the total number of firms / varieties, *n*, is always equal to the total supply of capital:

$$n = K \tag{3}$$

The number of varieties produced in period *t* in region *r* is:

$$n_{1,t} = \lambda_t n = \lambda_t K \qquad n_{2,t} = \left(1 - \lambda_t\right) n = \left(1 - \lambda_t\right) K \tag{4}$$

As with most economic geography models, the primary focus of the Footloose Capital model is the spatial location of manufacturing industry, governed here by the endogenous regional allocation of capital, λ_t, where $0 \leq \lambda_t \leq 1$.

In what follows, we complete the model by characterizing the short-run general equilibrium in period *t* contingent on λ_t, by specifying explicitly the capital migration process, and by analysing the long-run equilibrium given as fixed point of the capital mobility dynamics.

4. Short-run General Equilibrium

With the instantaneous establishment of equilibrium in the agricultural market and no transport costs, the agricultural price is the same in both regions. Since competition results in zero agricultural profits, the equilibrium nominal wage of workers in period *t* is equal to the agricultural product price and is therefore always the same in both regions. We take this wage / agricultural price as the *numeraire*. Under the assumption of identical behaviour, each firm sets the same local (mill) price p_r using the Dixit-Stiglitz pricing rule. Given that the wage is 1, the local price of every variety is:

(5) $$p_r = \frac{\beta_r \sigma}{\sigma - 1} \qquad p_1 = p \qquad p_2 = p\frac{\beta_2(H_2)}{\beta_1(H_1)} = ph$$

The effective price paid by consumers for one unit of a variety produced in the other region is $p_r T$.

Short-run general equilibrium in period *t* requires that each manufacturer meets the demand for its variety.[4] For a variety produced in region *r*:

(6) $$q_{r,t} = d_{r,t}$$

where $q_{r,t}$ is the output of each manufacturer in region *r* and $d_{r,t}$ is the demand for that manufacturer's variety. From equ. (5), the short-run equilibrium profit per variety in region *r* is:

(7) $$\pi_{r,t} = p_r q_{r,t} - \beta q_{r,t} = \frac{p_r q_{r,t}}{\sigma} = \left[\frac{\beta_r}{\sigma - 1}\right] q_{r,t}$$

[4] As a result of Walras' Law, equilibrium in all product markets implies equilibrium in the regional labour markets.

This profit per variety constitutes the regional rental per unit of capital.

Consumers face regional manufacturing price indices given by:

(8) $$\begin{aligned} G_{1,t} &= \left[n_{1,t} p_1^{1-\sigma} + n_{2,t} p_2^{1-\sigma} T^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = \left[\lambda_t + \left(1-\lambda_t\right)\phi z\right]^{\frac{1}{1-\sigma}} K^{\frac{1}{1-\sigma}} p \\ G_{2,t} &= \left[n_{1,t} p_1^{1-\sigma} T^{1-\sigma} + n_{2,t} p_2^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = \left[\lambda_t \phi + \left(1-\lambda_t\right) z\right]^{\frac{1}{1-\sigma}} K^{\frac{1}{1-\sigma}} p \end{aligned}$$

where $z \equiv h^{1-\sigma}$ in order to simplify the notation. Consumption per variety in each region is:

(9) $$\begin{aligned} d_{1,t} &= \left[M_1 G_{1,t}^{\sigma-1} + M_2 G_{2,t}^{\sigma-1} \phi\right] p_{1,t}^{-\sigma} \\ d_{2,t} &= \left[M_1 G_{1,t}^{\sigma-1} \phi + M_2 G_{2,t}^{\sigma-1}\right] p_{2,t}^{-\sigma} \end{aligned}.$$

M_r denotes the expenditures for manufactured goods in region *r*; M defines the world expenditures for manufactures $M = M_1 + M_2$ and $s_E = \frac{M_1}{M}$ its regional split. We see below that M_r, M and s_E are independent of λ_t. From equ. (6), (8) and (9)

(10) $$\begin{aligned} q_{1,t} = d_{1,t} &= \left[\frac{s_E}{\lambda_t + \left(1-\lambda_t\right)\phi z} + \frac{\left(1-s_E\right)\phi}{\lambda_t \phi + \left(1-\lambda_t\right) z}\right] \frac{1}{p} \frac{M}{K} \\ q_{2,t} = d_{2,t} &= \left[\frac{s_E \phi}{\lambda_t + \left(1-\lambda_t\right)\phi z} + \frac{1-s_E}{\lambda_t \phi + \left(1-\lambda_t\right) z}\right] \frac{1}{p z^{\frac{\sigma}{1-\sigma}}} \frac{M}{K} \end{aligned}$$

Therefore – see equ. (7) – short-run equilibrium profit per variety in region *r* is:

(11) $$\begin{aligned} \pi_{1,t} &= \left[\frac{s_{E,t}}{\lambda_t + \left(1-\lambda_t\right)\phi z} + \frac{\left(1-s_{E,t}\right)\phi}{\lambda_t \phi + \left(1-\lambda_t\right) z}\right] \frac{1}{\sigma} \frac{M}{K} \\ \pi_{2,t} &= \left[\frac{s_{E,t}\phi}{\lambda_t + \left(1-\lambda_t\right)\phi z} + \frac{1-s_{E,t}}{\lambda_t \phi + \left(1-\lambda_t\right) z}\right] \frac{z}{\sigma} \frac{M}{K} \end{aligned}$$

For future reference, note that regional and world profit incomes, $\Pi_{r,t}$ and Π respectively, are given by

(12) $$\Pi_{1,t} = \lambda_t K \pi_{1,t} \qquad \Pi_{2,t} = \left(1-\lambda_t\right) K \pi_{2,t} \qquad \Pi = \Pi_{1,t} + \Pi_{2,t} = \frac{M}{\sigma}$$

(for the latter use equ. (11)) and world gross income Y by

(13) $$Y = L + \frac{1}{\sigma} M \ .$$

The government plans a total level of public services of H. H_r is the fraction of public services allocated to region *r*. Providing one unit of *H* costs 1 (agricultural commodity is the numeraire), the total public expenditures are

(14) $$PE = PE_1 + PE_2 = H_1 + H_2 \, .$$

The government also decides upon the regional contribution to the financing of the public services with s_F denoting the share of region 1. Therefore the tax burdens of the two regions are

(15) $$TB_1 = s_F H \qquad TB_2 = \left(1 - s_F\right) H$$

Regional expenditures for manufactured goods are therefore given as

(16) $$M_1 = \mu\left(s_L L + s_K \Pi - TB_1\right) \qquad M_2 = \mu\left[(1 - s_L) L + (1 - s_K)\Pi - TB_2\right]$$

World expenditures for manufactures are

(17) $$M = \mu\left(L + \Pi - H\right) = \mu\left(L + \frac{M}{\sigma} - H\right).$$

Therefore,

(18) $$M = \frac{\mu}{\sigma - \mu} \sigma\left(L - H\right).$$

Its regional split is

(19) $$s_E = \frac{\left[s_L\left(\sigma - \mu\right) + s_K \mu\right] L - \left[s_F\left(\sigma - \mu\right) + s_K \mu\right] H}{\sigma\left(L - H\right)} \, .$$

World income (see equ. (13)), total expenditures for manufactures and its regional split are constant, i.e. independent of the regional allocation of capital.

With no provision of public services and no taxation, $H = 0$, region 1's share in total expenditure s_E is equal to

$$\overline{s_E} \equiv s_L \frac{\sigma - \mu}{\sigma} + s_K \frac{\mu}{\sigma}$$

When $\overline{s_E} \neq \frac{1}{2}$ factor endowments are unevenly distributed between the regions. In particular, when $\overline{s_E} < \frac{1}{2}$ region 1 is poorer (has a smaller factor endowment) than region 2.

With the provision of public services, $H > 0$:

(20) $$s_E > \overline{s_E} \quad \text{if} \quad s_F < s_L .$$

For region 1, the expenditure share for manufactured goods after taxation s_E will be greater than the expenditure share for manufactured goods before taxation $\overline{s_E}$, if consumers in region 1 contributes less than consumers in region 2 to the financing of the public services. If $s_F = s_L$ the expenditure share for manufactured goods is not affected by taxation.

Finally, equ. (11), (18) and (19) determine short-run equilibrium regional profits per variety.

Crucial for the subsequent dynamics is the relative profitability of capital $R(\lambda_t)$ given by:

(21) $$R(\lambda_t) = \frac{1}{z} \frac{\left[\lambda_t \phi + (1-\lambda_t) z\right] + (1-s_E)\phi\left[\lambda_t + (1-\lambda_t)\phi z\right]}{s_E \phi\left[\lambda_t \phi + (1-\lambda_t) z\right] + (1-s_E)\left[\lambda_t + (1-\lambda_t)\phi z\right]}.$$

For a constant s_E the relative profitability of capital depends upon the allocation of capital λ_t via the so-called "competition effect": a higher λ_t increases the competition in region 1 and therefore reduces relative profitability. The competition effect implies a negative slope of $R(\lambda_t)$, i.e. $\frac{\partial R(\lambda_t)}{\partial \lambda_t} < 0$.

5. Capital Movements and the Complete Dynamical Model

In a Footloose Capital model, the representative capitalist does not move herself, but allocates the physical capital she owns between the regions. In doing so, she is interested in her *real net* income (we assume that she takes the level of the publicly provided services at home as given). Since all income is taxed and spent in the home region of the capitalist, the relevant tax rate and price index for calculating *real net* income are the ones at home, irrespective of the regional capital allocation. Therefore, in this case location choices based on *real net* income and on *nominal gross* income are equivalent.

The concrete specification of the dynamic process follows ideas from the replicator dynamics widely used in evolutionary economics and evolutionary game theory (see e.g., Weibull, 1997; see also Fujita et al., 2000, p. 77, who points to this fact). Taking into account the constraint $0 \leq \lambda_{t+1} \leq 1$, the piecewise smooth one-dimensional map whereby λ_{t+1} is determined by λ_t is:

$$\lambda_{t+1} = Z(\lambda_t) = \begin{cases} 0 & if \quad F(\lambda_t) < 0 \\ F(\lambda_t) & if \quad 0 \leq F(\lambda_t) \leq 1 \\ 1 & if \quad F(\lambda_t) > 1 \end{cases} \tag{22}$$

where λ_t is in [0,1] implies that λ_{t+1} is in [0,1] and where

$$\frac{F(\lambda_t) - \lambda_t}{\lambda_t} = \gamma E_t = \gamma \frac{\pi_{1,t} - (\lambda_t \pi_{1,t} + (1-\lambda_t)\pi_{2,t})}{\lambda_t \pi_{1,t} + (1-\lambda_t)\pi_{2,t}}. \tag{23}$$

We refer to $\gamma > 0$ as the "speed" with which the representative capitalist alters the share of capital in region 1 in response to economic incentives E_t, in particular to a comparison of the rate of profit in region 1 with the average rate of profit, given by $(\lambda_t \pi_{1,t} + (1-\lambda_t)\pi_{2,t})$. It can be transformed into a law of motion depending upon the ratio in regional profitability, $R(\lambda_t)$:[5]

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[5] Note that – from an analytic perspective – this specification is a good approximation to the discrete-time counterpart of the process assumed by Puga (1998) in his core-periphery model.

$$F\left(\lambda_t\right)=\lambda_t+\gamma\lambda_t\left(1-\lambda_t\right)\frac{R\left(\lambda_t\right)-1}{\lambda_t R\left(\lambda_t\right)+\left(1-\lambda_t\right)}. \tag{24}$$

Fixed points for the dynamical system, which correspond to points of rest or long-run equilibria, are defined by $Z\left(\lambda\right)=\lambda$. Core-periphery equilibria, i.e. $\lambda_0^{CP}=0$ or $\lambda_1^{CP}=1$, are boundary fixed points of the dynamic system. A central question of the New Economic Geography concerns critical values for trade freeness (or for any other parameter) at which agglomeration in either region is sustainable. The so-called sustain points give conditions under which "the advantages created by such a concentration, should it somehow come into existence, [are] sufficient to maintain it" (Fujita et al., 2000, p. 9). Sustain points therefore specify conditions at which the boundary equilibria λ_i^{CP} (where i = 0, 1) become (at least locally) stable. These critical values are defined by $F'\left(\lambda_i^{CP}\right)=1$, with the latter indicating the derivative of the first return map equ. (23). The latter condition can be reduced to $R\left(\lambda_i^{CP}\right)=1$ and solved for

$$\phi_{1,2}^{S(0)}=\frac{z\pm\sqrt{z^2-4s_E\left(1-s_E\right)}}{2\left(1-s_E\right)} \qquad \phi_{1,2}^{S(1)}=\frac{1\pm\sqrt{1-4z^2 s_E\left(1-s_E\right)}}{2zs_E}, \tag{25}$$

where $\phi^{S(i)}$ indicates the sustain point for λ_i^{CP}. Tables 1 and 2 and figure 6 summarize the properties of the sustain values.

In addition to the boundary fixed points, an interior fixed point is given by

$$\lambda^*=\frac{1}{2}+z\frac{(1-\phi)(1+\phi)}{(1-z\phi)(z-\phi)}\left(s_E-\frac{1}{2}\frac{(1+z\phi)(z-\phi)}{(1-\phi)(1+\phi)z}\right). \tag{26}$$

Note that the condition $\phi<z<\phi^{-1}$ is necessary for $0<\lambda^*<1$ to hold.[6] That is, for an interior equilibrium to exist, the two regions should not differ too much in terms of provisions of public services within their territory.

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[6] This can be easily verified considering that $0<\lambda^*<1$ for $\phi\frac{z-\phi}{1-\phi^2}<s_E<\frac{1}{z}\frac{z-\phi}{1-\phi^2}$. Indeed, a necessary condition for these inequalities to hold is $\phi<z<\phi^{-1}$.

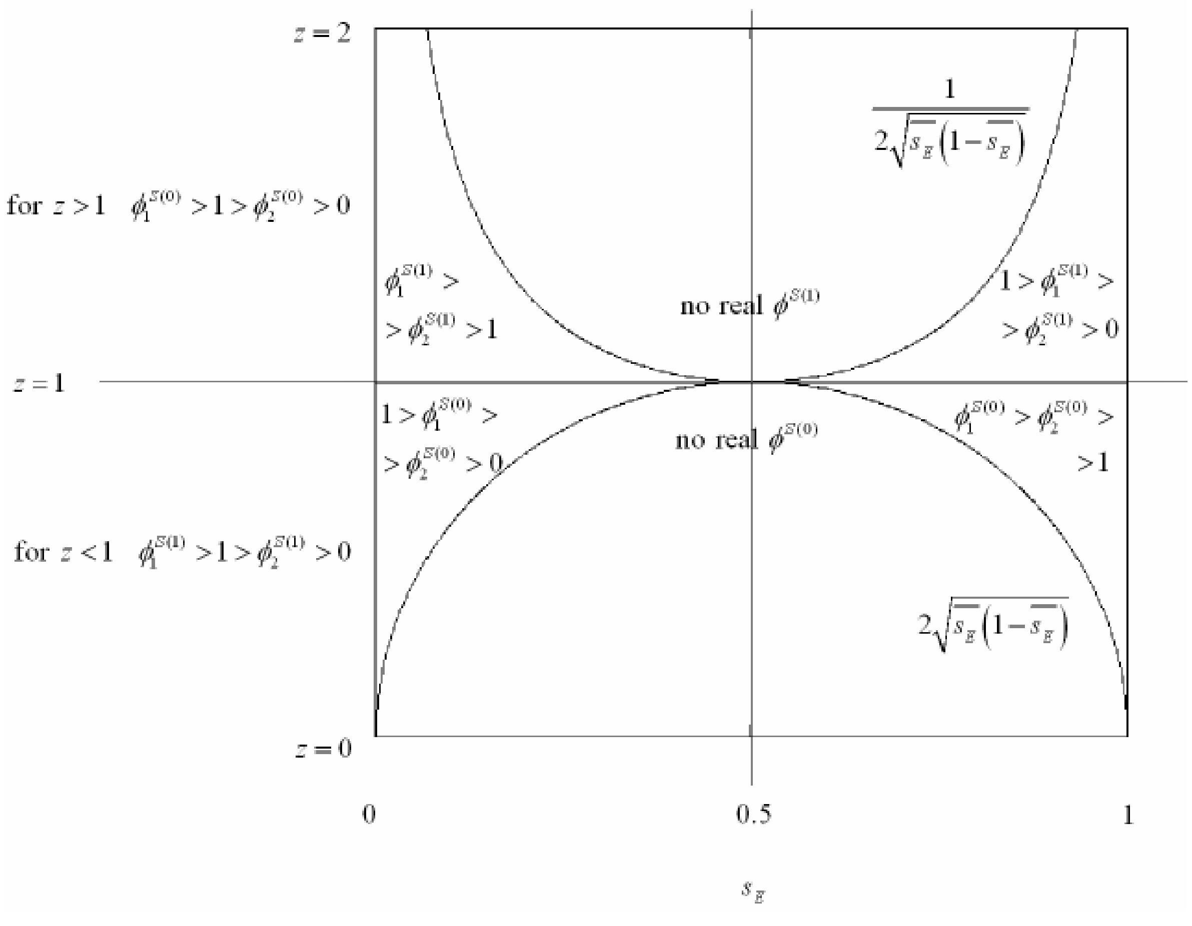
Table 1: Properties of the sustain values for the boundary fixed point $\lambda_0^{CP} = 0$

<table>
<tr><th colspan="3">Properties of $\phi_{1,2}^{S(0)}$</th></tr>
<tr><td>$1 < z$</td><td colspan="2">$\phi_{1,2}^{S(0)}$ are both real and $0 < \phi_2^{S(0)} < 1 < \phi_1^{S(0)}$ holds</td></tr>
<tr><td rowspan="2">$2\sqrt{s_E\left(1-s_E\right)} < z < 1$</td><td colspan="2">$\phi_{1,2}^{S(0)}$ are both real</td></tr>
<tr><td>$s_E < 0.5:\ 0 < \phi_2^{S(0)} < \phi_1^{S(0)} < 1$</td><td>$s_E > 0.5:\ 1 < \phi_2^{S(0)} < \phi_1^{S(0)}$</td></tr>
<tr><td>$z < 2\sqrt{s_E\left(1-s_E\right)}$</td><td colspan="2">No real $\phi_{1,2}^{S(0)}$ exists</td></tr>
</table>

Table 2: Properties of the sustain values for the boundary fixed point $\lambda_1^{CP} = 1$

<table>
<tr><th colspan="3">Properties of $\phi_{1,2}^{S(1)}$</th></tr>
<tr><td>$1 < \dfrac{1}{2\sqrt{s_E\left(1-s_E\right)}} < z$</td><td colspan="2">No real $\phi_{1,2}^{S(1)}$ exists</td></tr>
<tr><td rowspan="2">$1 < z < \dfrac{1}{2\sqrt{s_E\left(1-s_E\right)}}$</td><td colspan="2">$\phi_{1,2}^{S(1)}$ are both real</td></tr>
<tr><td>$s_E < 0.5:\ 1 < \phi_2^{S(1)} < \phi_1^{S(1)}$</td><td>$s_E > 0.5:\ 0 < \phi_2^{S(1)} < \phi_1^{S(1)} < 1$</td></tr>
<tr><td>$z < 1$</td><td colspan="2">$\phi_{1,2}^{S(1)}$ are both real and $0 < \phi_2^{S(1)} < 1 < \phi_1^{S(1)}$ holds</td></tr>
</table>

Figure 6: Properties of the sustain values for the boundary fixed points $\lambda_0^{CP} = 0$ and $\lambda_1^{CP} = 1$



A second central question of the New Economic Geography concerns critical values for the trade freeness (or for any other parameter) at which an (interior) equilibrium without spatial concentration "breaks up". This so-called break point gives conditions under which "small differences among locations [will] snowball into larger differences over time, so that the symmetry between identical locations will spontaneously break" (Fujita et al., 2000, p. 9). I.e. it gives conditions under which an interior fixed point λ^* becomes (at least locally) unstable and the dynamics is attracted to one of the boundary equilibria. Analytically, the break point is defined by $F'\left(\lambda^*\right)=1$. In our model, the break point ϕ^B arises when the interior fixed point coincides with one of the boundary fixed points and it is equal to the corresponding sustain point. At that value of the trade freeness a transcritical bifurcation occurs (see Wiggins, 1990). Two fixed points (that is, the interior fixed point and one of the boundary fixed points) cross each other (with the interior fixed point leaving the admissible interval) and they exchange stability.

The fixed point may lose stability via a Flip bifurcation, defined by $F'\left(\lambda^*\right)=-1$; it occurs if the parameters satisfy the following condition:

$$1+\gamma\frac{\left(\phi\left(z-\phi\right)-s_E\left(1-\phi^2\right)\right)\left(z-\phi-zs_E\left(1-\phi^2\right)\right)}{zs_E\left(1-s_E\right)\left(1-\phi^2\right)^2}=-1\,. \tag{27}$$

Figure 7 presents bifurcation diagrams showing the impact of trade freeness ϕ on the long-term regional allocation of capital λ_t for (a) z = 1 and (b) z = 0.98. The other parameters are set at the values $s_E=0.4$ and $\gamma=10$ and the initial condition is close to the interior fixed point value $\lambda_0=1.005\lambda^*$.

Figure 7: Bifurcation diagram showing the impact of trade freeness ϕ on the long-run behaviour of region 1's share of capital use λ_t for (a) $z=1$ and (b) $z=0.98$.

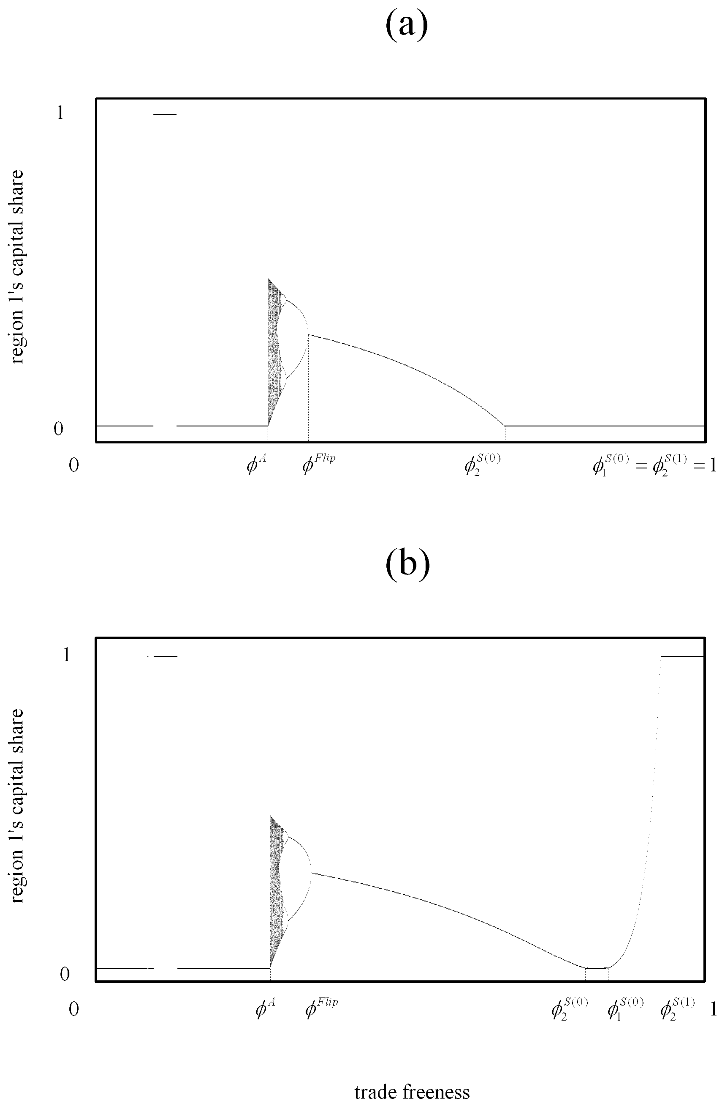


Figure 7(a) is plotted for z = 1. Since $s_E < 0.5$, from the properties of the sustain points (see Table 2 above),[7] it follows:[8]

$$\phi_2^{S(0)} = \frac{s_E}{1-s_E} < \phi_1^{S(0)} = \phi_2^{S(1)} = 1 < \phi_1^{S(1)}$$

The boundary equilibrium $\lambda_1^{CP} = 1$ is (locally) unstable for all values of ϕ. In an economy highly integrated (i.e. low transport costs or high trade freeness), in particular for $\phi_2^{S(0)} < \phi < 1$ no interior fixed point exists within the (0, 1) interval, and the boundary equilibrium $\lambda_1^{CP} = 0$ is locally stable. As ϕ crosses the sustain point $\phi_2^{S(0)}$, a transcritical bifurcation occurs: $\lambda_1^{CP} = 0$ loses stability; the interior fixed point enters the (0, 1) interval and becomes locally stable. At intermediate values of trade freeness, that is, for $\phi^{Flip} < \phi < \phi_2^{S(0)}$, the interior fixed point λ^* is (locally) stable. Looking at this interval it can be noticed that, increasing ϕ, λ^* declines. The larger market size favours agglomeration in region 2.

With stronger trade barriers, as ϕ crosses ϕ^{Flip}, the interior fixed point loses stability via a Flip bifurcation. Attracting periodic solutions appear, first a period two cycle, then – at lower values of ϕ – the time path exhibits cycles of any order and even chaotic patterns with an ever increasing volatility of the regional shares of capital. For $\phi < \phi^A$, the volatility that results for relatively high trade costs leads to the concentration of all industrial activity in one of the regions. Given the mobility hypothesis as specified in (22), the share of capital does not longer change once one of the boundary values 0 or 1 is reached. A core-periphery outcome emerges even though both boundary fixed points are locally unstable.[9]

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[7] Since they coincide with the sustain points, we do not mark the break points in the Figures .

[8] From z = 1 and $s_E < 0.5$, it follows $\lambda^* = \frac{1}{2} + \frac{1+\phi}{1-\phi}\left(s_E - \frac{1}{2}\right)$ and $\frac{\partial \lambda^*}{\partial \phi} = -\frac{1-2s_E}{(1-\phi)^2} < 0$. Given that $\lambda^* = s_E$ at $\phi = 0$ and $\lambda^* \to -\infty$ for $\phi \to 1$, the interior equilibrium curve λ^* in Figure 2(a) cuts necessarily the 0 line when $0 < \phi < 1$.

[9] A more detailed account of the phenomenon of agglomeration via volatility for the standard Footloose Capital model with no government sector is presented in Commendatore, Currie and Kubin (2007).

Figure 7(b) is plotted for $z = 0.98$. Since $2\sqrt{s_E\left(1-s_E\right)} < z < 1$, from the properties of the sustain values, it follows:[10]

$$\frac{s_E}{1-s_E} < \phi_2^{S(0)} < \phi_1^{S(0)} < \phi_2^{S(1)} < 1 < \phi_2^{S(2)} .$$

For $0 \le \phi \le \phi_2^{S(0)}$ the behaviour of λ_t in (b) is qualitatively similar to the behaviour in (a). The most notable changes occur in the interval $\phi_2^{S(0)} < \phi < 1$. Firstly, full agglomeration in region 2 takes place at a higher value of the sustain point $\phi_2^{S(0)}$.

Secondly, even though the boundary fixed point $\lambda_0^{CP} = 0$ is stable for $\phi_2^{S(0)} < \phi < \phi_1^{S(0)}$ as in (a), it loses its stability with increasing economic integration. As ϕ crosses the sustain point $\phi_1^{S(0)}$ from left to right, a transcritical bifurcation occurs: the interior fixed point re-enters the (0, 1) interval and becomes locally stable. Looking at this interval it can be noticed that, increasing ϕ, λ^* increases as well. With increasing economic integration, the productivity rise in region 1 – induced by a larger provision of public services – overcomes the disadvantage of a smaller market size and favours agglomeration in this region.

Finally, increasing further ϕ, as ϕ crosses the sustain point $\phi_2^{S(1)}$ another transcritical bifurcation occurs; the interior fixed point exits the (0, 1) interval, losing stability; and the boundary fixed point $\lambda_0^{CP} = 1$ gains local stability. On the contrary of the previous case,

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[10] For $z < 1$ and $s_E < 0.5$, the interior equilibrium curve in figure 2(b) has a relative minimum at $\phi_{\min}$ and a relative maximum at $\phi_{\max}$, where $0 < \phi_{\min} < z < \phi_{\max} < 1$ and where

$$\phi_i = \frac{z(1-2s_E) \pm (1-z)\sqrt{s_E(1-s_E)}}{1-(1+z^2)s_E} \text{ and } i = \max, \min$$

When $z > 2\sqrt{s_E\left(1-s_E\right)}$, the interior equilibrium curve – which at $\phi = 0$ starts from $\lambda^* = s_E$ – cuts the 0 line from above. This is because its (relative) minimum is negative, $\lambda^* < 0$ at $\phi_{\min}$. The curve then cuts the 0 line from below and after the 1 line from below before leaving the interval (0, 1). This is because $\lambda^* \to \infty$ for $\phi \to z$.

strong economic integration determines a "near catastrophic" agglomeration of capital from region 2 to region 1.

The differences between (a) and (b), which are more remarkable for $\phi_2^{S(0)} < \phi < 1$, follow from a small change in the parameter *z*. The latter depends on H_1 and H_2 – the regional provisions of public services – that, in our model, are crucial policy tools available to the central government, together with s_F and $\left(1-s_F\right)$, the regional contributions to the financing of public services. The parameter s_F affects region 1' expenditure share for manufactured goods and therefore the regional relative market sizes.

We leave further considerations upon public policies to the next section.

6. The impact of public expenditure on the industrial location

In order to understand the impact of public expenditure on the concentration of industrial activity in our model, we first consider the interior equilibrium λ^*. The provision of public services in region 1 affects λ^* as follows:

(28) $$\frac{\partial\lambda^*}{\partial H_1} = -\frac{\phi}{(1-\phi z)^2}\left[1-\frac{(z^2-1)(1-\phi^2)}{(z-\phi)^2}s_E\right]\frac{\partial z}{\partial H_1} + z\frac{1-\phi^2}{(1-\phi z)(z-\phi)}\frac{\partial s_E}{\partial H_1}$$

It is possible to identify neatly two effects that an increase in H_1 could exert on the location of the manufacturing sector. According to the 'productivity effect', expressed by the first term in equation (28)

$$-\frac{\phi}{(1-\phi z)^2}\left[1-\frac{(z^2-1)(1-\phi^2)}{(z-\phi)^2}s_E\right]\frac{\partial z}{\partial H_1},$$

the provision of public services in region 1 affects λ^* via its effect on the labour productivity in the manufacturing sector located in region 1. Since the term in the square brackets is positive for any $0 \le s_E \le 1$ and $\frac{\partial z}{\partial H_1} < 0$, the productivity effect is positive on λ^*.

According to the "demand effect", expressed by the second term in equation (28)

$$z \frac{1-\phi^2}{(1-\phi z)(z-\phi)} \frac{\partial s_E}{\partial H_1},$$

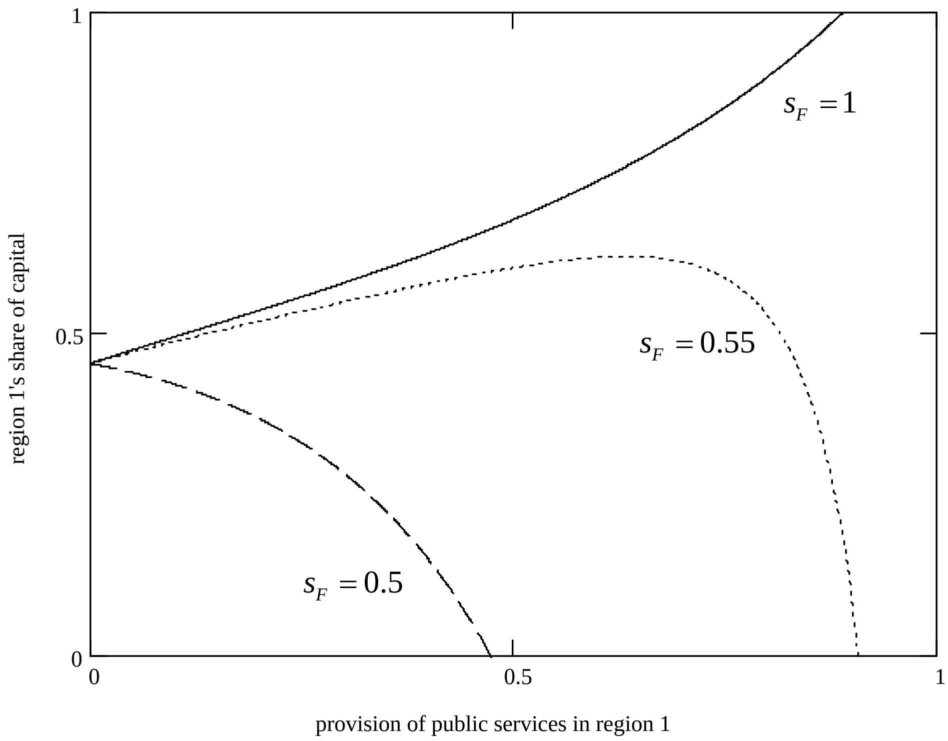
the provision of public services in region 1 affects λ^* via a change in the relative market size. Since $\phi < z < \phi^{-1}$ and $\phi < 1$, the sign of the demand effect corresponds to the sign of $\frac{\partial s_E}{\partial H_1} = (s_L - s_F)\frac{\sigma - \mu}{\sigma}\frac{L}{(L-H)^2}$. Therefore, for $s_F > s_L$, the demand effect is negative on λ^*. Conversely, for $s_F < s_L$, the demand effect is positive on λ^*. For $s_F = s_L$ no demand effect occurs.

In order to disentangle the relative importance of these two effects, in the following analysis we employ numerical simulations and analyse the following stylised case: region 1 is the 'poor' region, in the sense of having the lower income share without any public expenditures. In order to improve the situation in region 1, public expenditures that enhance productivity in region 1 are increased. We study whether and how the effect of such a policy depends upon its financing scheme. We set $s_L = 0.5$, $s_K = 0.25$, $\sigma = 4$ and $\mu = 0.5$, from which it follows $\overline{s_E} = 0.46875$. That is, since $\overline{s_E} < 0.5$, region 1 is the poor region since it has a smaller factor endowment than region 2. We also set $\phi = 0.2$, $L = 1$, $A = 1$, $B = 0.45$ and $H_2 = 0$, i.e. region 2 receives no public expenditures.

Figure 8 summarises the effects on λ^* of an increase in the provision of public services in region 1, within the interval $0 \le H_1 < L$,[11] for different values of the regional tax burden necessary to finance it, s_F. The solid line corresponds to $s_F = s_L = 0.5$, that is, the burden of taxation is equally distributed among consumers in the two regions. The demand effect is nil and only the productivity effect impact (positively) on region 1's share of capital.

[11] H_1 should be smaller than L in order to have positive world expenditures for manufactures, $M > 0$.

Figure 8: Diagram showing the impact of the provision of public services in region 1 on the interior equilibrium λ^* for different values of the share of the tax burden necessary to finance public services.



The dotted line corresponds to $s_F = 0.55$. The demand effect is negative and after an initial range it overcomes the productivity effect.[12] Finally, the dashed line corresponds to $s_F = 1$, the demand effect is stronger than the productivity effect for any H_1 in the interval considered.[13]

The latter result tells us that policy measures aimed at enhancing labour productivity of manufacturing firms in the backward region will be effective only if the prosperous region participates to the financing of such policies. If the poor region is “left alone” (that is, if public services are financed solely by income of residents in that region – corresponding to $s_F = 1$ in our simulation study), then the demand effect of an increase in H_1 will eventually prevail. On the other hand, if the government sets s_F at a sufficiently small value, letting region 2 tax payers contribute on the basis of their capacity (s_F equal to s_L or smaller – in our simulation study we explored the case $s_F = s_L = 0.5$), then the demand effect of public expenditures is negligible or it could even act in the same direction of the productivity effect.

We now consider the impact of the provision of public services on the long term behaviour of region’s 1 share of capital λ_t. Figure 9 confirms the above conclusions. That is, increasing H_1 could favour agglomeration in region 1 as long as such increase is not too large. Otherwise the demand effect could overcome the productivity effect inducing agglomeration of the manufacturing sector in region 2. As shown in figure 9(a), plotted for $\phi = 0.3$, for $0 < H_1 < H_{1,2}^{S(1)}$ increasing the provision of public services favours the location

[12] Note that whereas the demand effect tends to infinity for $H \to L$, the productivity effect is positive but finite.

[13] However, note that for $H_2 < H_1$ ($z < 1$) and $\phi < z < \phi^{-1}$ a higher trade freeness strengthens the productivity effect and weakens the demand effect. Therefore, by increasing ϕ, there could be an initial range of values of H_1 for which λ^* increases even for $s_F = 1$.

of the industrial activity in region 1.[14] For $H_1^{Flip} < H_1 < H_{1,2}^{S(1)}$, the interior fixed point is locally stable, whereas for $0 < H_1 < H_1^{Flip}$ complex behaviour takes place in the long run. [15] As H_1 crosses $H_{1,2}^{S(0)}$ a transcritical bifurcation takes place: the interior fixed point leaves the (0, 1) interval, losing stability; and the boundary fixed point $\lambda_0^{CP} = 1$ becomes locally stable. Compared with the behaviour of λ^* in Figure 8, for which $\phi = 0.2$, a higher value of ϕ determines full agglomeration of the industrial activity in region 1 by strengthening the productivity effect.

Increasing further H_1, however, reinforces the demand effect offsetting eventually the productivity effect. As H_1 crosses $H_{1,1}^{S(1)}$, the boundary fixed point, $\lambda_0^{CP} = 1$ loses stability; the interior fixed point re-enters the (0, 1) interval and gains local stability. The increase in taxation necessary to finance the provision of public services reduces substantially region 1's market size. Therefore while H_1 rises, the manufacturing sector shift progressively in region 2. Finally, as H_1 crosses the sustain point $H_{1,1}^{S(0)}$ another transcritical bifurcation takes place; the interior fixed point exits the (0, 1) interval, losing stability; and the boundary fixed point $\lambda_0^{CP} = 0$ gains stability. All manufacture migrates in region 2 through a 'near catastrophic' process of agglomeration.

Figure 9(b), which is plotted for a smaller value of trade freeness (i.e., $\phi = 0.23$), differs from figure 9(a) in two important respects: First, the productivity effect is weaker and the demand effect stronger. Therefore, when the provision of public services is relatively large, full agglomeration in region 1 never occurs. Moreover, full agglomeration in region 2 takes place through a smoother process.

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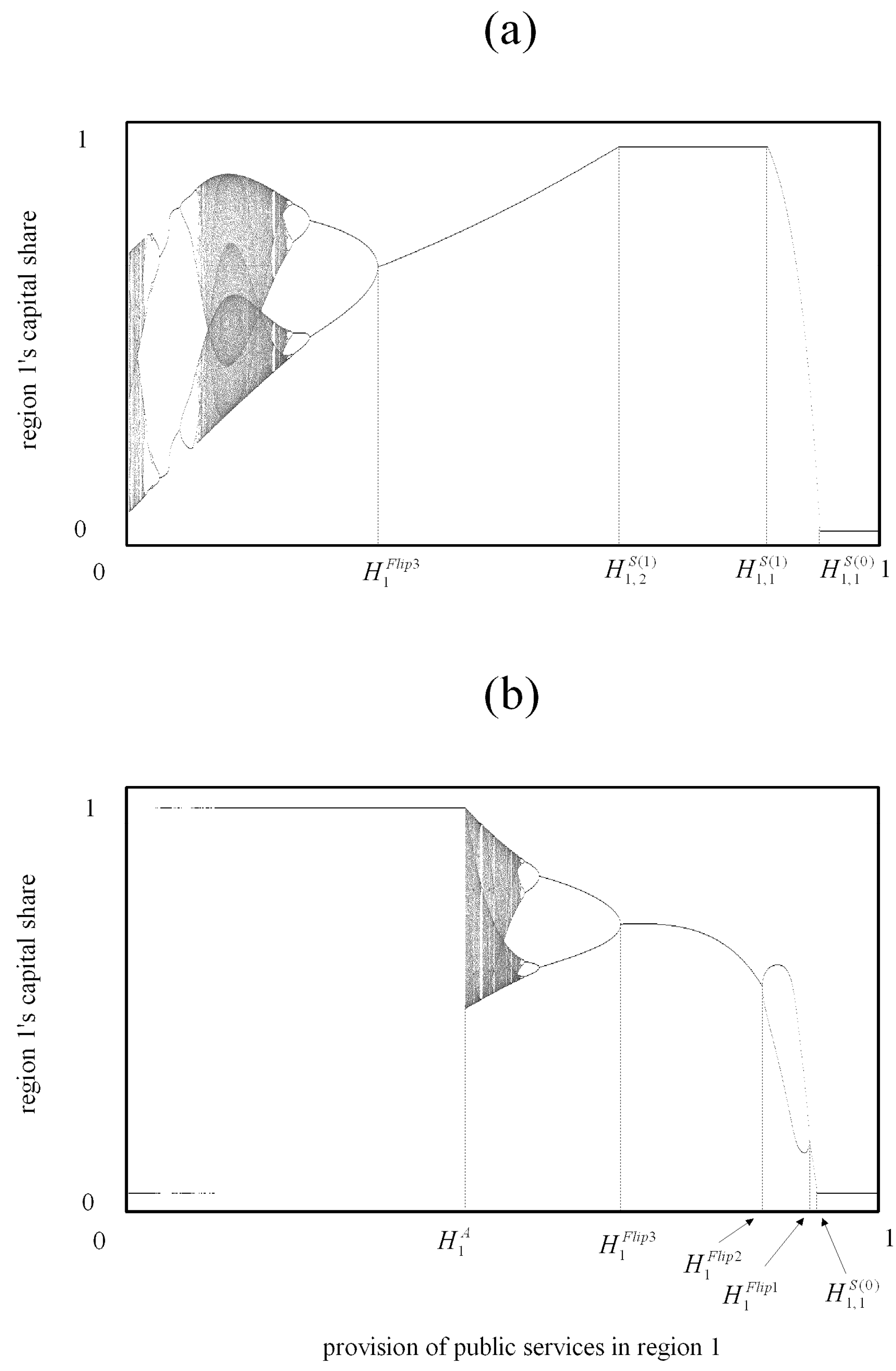
[14] For H_1 there can be a maximum of three sustain points (and break points): two for the boundary equilibrium $\lambda_0^{CP} = 0$ and two for $\lambda_1^{CP} = 1$. In figure 4(a) three of them are visible, $H_{1,1}^{S(0)}$, $H_{1,1}^{S(1)}$ and $H_{1,2}^{S(1)}$; whereas in figure 4(b) only one, $H_{1,1}^{S(0)}$.

[15] There are four values of H_1 which satisfies condition (27). In figure 4(a) only one of them is visible, H_1^{Flip}; whereas in figure 4(b) three of them are, H_1^{Flip1}, H_1^{Flip2} and H_1^{Flip3}.

Figure 9: Bifurcation diagram showing the impact of the provision of public services in region 1 H_1 on the long-run behaviour of region 1's share of capital use λ_t for (a) $\phi = 0.3$ and (b) $\phi = 0.23$.



Second, the long-term behaviour of the capital shares is considerably more volatile. The first Flip bifurcation value H_1^{Flip3} is much higher. Moreover, within the interval $H_1^{Flip3} < H_1 < H_{1,1}^{S(0)}$, other two Flip bifurcations occur for the interior fixed point. Finally, for $0 \leq H_1 < H_1^A$ the volatility of region 1's capital share is so high that, sooner or later, it converges on either $\lambda_0^{CP} = 0$ or $\lambda_1^{CP} = 1$.[16]

7. Conclusions

This paper delivers further insights on the impact of productive public spending on industrial location, extending results provided by Commendatore et al. (2007) in two main directions: confirming their conclusions in a much easier framework and studying the dynamic behaviour of relevant economic variables.

We have dealt with a FC model without investment goods and with a government using the agricultural good as an input in the provision of public services. Our main contributions are the following.

First, we have fully characterized the dynamics of capital movements in the model, looking at stability properties of industrial location equilibria, under alternative degrees of economic integration.

Second, we have decomposed the overall effect of an increase in the endowment of productivity enhancing public services available to firms located in backwards regions into two component: the productivity effect and the demand effect. That is, firms are attracted by lower input requirements, while higher taxation tend to shrink the local market, leading firms to relocate elsewhere. As in Commendatore et al. (2007) the demand effect will be nil only if the tax burden is equally distributed across regions. On the other hand, the demand effect of public expenditures will be offset by (or even act in the same direction of) the

[16] In this interval, the long-term location of the overall manufacturing sector is highly sensitive to the initial condition λ_0. See Commendatore, Currie and Kubin (2007).

productivity effect if the government let the tax payers of the richer region contribute on the basis of their capacity.

Third, the above policy analysis has been extended to a dynamic context, studying the long term regional relocation of capital induced by public spending, letting varying the degree of economic integration. In doing so, we have drawn a full picture of the relative strength of the productivity and the demand effects under alternative scenarios of trade freeness.

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