Composition of Public Expenditure, effective demand, distribution and growth

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1. Introduction

This paper deals with the role of government expenditure and its composition in Post Keynesian analysis. This subject has been largely overlooked by this tradition of thought, in spite of the fact that some of its founders paid attention to it. As Pressman (1994) notices, Keynes underlined that there are economic and political reasons for preferring certain kinds of expenditures to others. Kaldor (1958, pp. 136-137; 1966; 1967; and 1971) pointed out that the composition of government expenditure has important effects on long-run growth. For him, government consumption can transform the economy into one with low investment, with some undesirable consequences on its international competitiveness and long-run growth due to the fact that the capital goods sector tends to enjoy higher rates of variation in productivity than the consumption goods sector.

The views of Keynes and Kaldor, which provide interesting insights into the complexity of the growth processes, were not presented in a formalised way, nor have they been subsequently formalised by other authors. A formal treatment of the role of government expenditure in the analysis of growth only refers to the aggregate level of this variable (see You and Dutt, 1996; Lavoie, 2000; Commendatore, Panico and Pinto, 2005). It does not deal with its composition.

A more detailed account of the effects of government expenditure on growth can be found in the tradition of thought following the lines set by Barro (1990), whose analyses too have mainly focussed on the relationship between the size of the government sector and economic growth. According to Devarajan et al. (1996, p. 314), 'Much less is known about how the composition of public expenditure affects a country's growth rate', in spite of the fact the mix between what is called "productive" and "unproductive" government expenditure can affect the performance of the economy. Barro (1990) assumes that government expenditure is complementary with private production and enter the production function. It has two opposite effects on the rate of growth, one positive, working through the increase in the productivity of private capital, and one negative, working through the reduction of saving due to the increase in tax revenues. The analyses dealing the composition of government expenditure assume that both "productive" and "unproductive" expenditures enter the production function and affect the productivity of the private sector and conclude that these expenditures can influence positively or negatively the rate of growth according to their relative share of government spending (see Devarajan et al., 1994, pp. 317-318). This result is due to the assumptions that the two kinds of expenditures are complement in the government balance, whose dimension is taken as given, and enter the production function by generating diminishing marginal returns.

The economic mechanisms captured by these analyses only refer to the effects on the rate of growth emerging in the production or supply side of the economy. Those produced by the variations in income distribution and effective demand are totally absent.

In what follows an attempt is made to develop an analysis that also takes into account the effects on the rate of growth of variations in income distribution and effective demand produced by changes in government expenditure and its composition. Like in Barro (1990) and in the literature recalled above, here too government expenditure can affect the coefficients of production, and therefore inputs' productivity. Moreover, the increase in productivity does not necessarily lead to an increase in the rate of growth of the economy. Yet, in this analysis here presented, this result is not due to the assumption that the two kinds of expenditures enter the production function by generating diminishing marginal returns. It is due instead to other effects, which are absent in Barro's analysis, generated by the variations in income distribution and effective demand. These effects can work in different ways. When government expenditure is "unproductive" (i.e. when it does not affect the coefficients of production), its variation causes a transfer of income from the private to the Government sector. When it is "productive" (i.e. when it affects the coefficients of production) its variation can also cause a re-distribution of income between capitalists and workers, depending on how the increase in productivity is appropriated by the two classes. In both cases, a change in effective demand occurs due to the fact that the propensity to consume of the capitalist class is smaller than the propensity to spend of the government sector and the propensity to consume of the working class. This change causes in turn a variation in the rate of growth of the economy.

The analysis presented below develops a model in which the government sector works with a balanced budget. Its expenditure can affect the coefficients of production, and hence inputs' productivity, and generate further effects on income distribution, savings and effective demand. The investment function is assumed to be non-linear. This assumption, as will be seen, allows the model to reproduce a variety of complex phenomena, like multiple equilibria, hysteresis, low growth traps, and regular and irregular growth cycles.

The model can be interpreted along Kaleckian and Classical Harrodian lines. The first interpretation considers the state of long term expectations of investors as exogenously given, driven for instance on entrepreneurs' animal spirits. The second considers that investor's expectations are related to the "warranted" rate of growth, in the sense that the expected level of demand and output of the economy is the one corresponding to the "warranted" rate of growth. The two assumptions on the formation of expectations generate a different set of results on the way in which "productive" and "unproductive" government expenditures affect the rate of growth. These results are summarised in the conclusions.

The paper is so organised. Section 2 presents the basic model. Section 3 deals with its Kaleckian interpretation showing how "productive" and "unproductive" government expenditures can affect the rate of growth within this framework. Section 4 deals with the Classical-Harrodian interpretation of the model and its results. Section 5 presents the conclusions summarising the main results.

2. The model

We study a single-good closed economy. Technical progress is excluded. The production function is of a Leontief type with two factors of production, labour and fixed capital. The labour supply is perfectly elastic and capital does not depreciates. Hence:

$$Y^P = aK \quad and \quad Y = bL \tag{1}$$

where Y^P is the potential output, Y the current output, K the stock of capital, L the amount of labour employed in production, a and b are the reciprocal of the capital and labour coefficients respectively.

In each period the capacity of the capital stock is not fully utilised, so that the potential and current outputs do not coincide. The degree of capacity utilization is defined as:

$$u = \frac{Y}{Y^P} \tag{2}$$

Income is distributed between wages and profits: $Y = wL + rK$, where w is the wage rate and r is the rate of profit. Normalising with respect to output, and taking into account expressions (1) and (2) this equation becomes

$$1 = \frac{w}{b} + \pi \tag{3}$$

where w/b and $\pi \equiv r/au$ are respectively the share of wages and the share of profits in national income. We assume that the wage rate is a function of labour productivity:

$$w = w(b) \quad with \quad w'(b) \geq 0$$

where the value of $w'(b)$ depends on the bargaining power of unions. Letting

$$w = w_0 b^{\lambda} \tag{4}$$

the wage-productivity elasticity $\lambda \geq 0$ measures the ability of unions to capture labour productivity improvements. When $0 \leq \lambda < 1$ workers are not able to fully capture the increase in productivity; and when $\lambda \geq 1$ wage increases are equal or higher than productivity improvements.

The saving function assumes that workers consume all their income:

$$s = s_{\pi} r(1-\tau) \tag{5}$$

Notice that s is the ratio saving to capital, that s_{π} is the propensity to save out of profits and τ is the tax rate.

We assume a non-linear investment function:[1]

$$g = \alpha + \phi(u) \tag{6}$$

According to equation (6), capital accumulation g depends on a component α, which is independent of u, and on the degree of capacity utilisation in a nonlinear way. The non-linear component has the following properties:

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[1] For simplicity, we did not incorporate the rate of profit into the investment function. Our result are not substantially modified, if we were to make such an assumption.

$$\phi(\tilde{u}) = 0, \quad \phi' > 0, \quad and \quad \phi'' \geq (<)\, 0 \quad for \quad u \leq (>)\, \tilde{u} \tag{7}$$

where $0 < \tilde{u} \leq 1$ is the *normal* degree of capacity utilization that is interpreted as the optimal degree of capacity utilization given the existing technology. Therefore, investment increases with the current degree of capacity utilisation. However, due to non linearity of this function, the speed of capital accumulation changes in the following way: when the degree of utilisation is substantially below the normal value, capital accumulation is slow since the entrepreneurs incentive for capital accumulation is small; when the degree of utilisation is substantially above the normal value, investment slows down again, due to the increasing installation costs and to the increasing risks of entering other projects and of getting additional financial resources with the existing net wealth.

The component α of the investment function can be interpreted along Kaleckian and Classical-Harrodian lines. In the first case, α is interpreted as an expression of entrepreneurs' animal spirits and is taken as exogenously given in the same way as in Keynesian models the state of long term expectations is taken as given (see Amadeo, 1986; and Lavoie 1996):

$$\alpha = \overline{\alpha} \tag{8}$$

In the second case, α coincides with the Harrodian 'warranted rate of growth', which depends on the saving generated at normal capacity utilisation:

$$\alpha = \tilde{g} \equiv s_{\pi}\tilde{r}(1-\tau) \tag{9}$$

where $\tilde{g}$ is the warranted rate of growth and $\tilde{r} \equiv \pi a \tilde{u}$ is the rate of profit corresponding to normal capacity utilisation.

We assume a balanced government budget:

$$\tau = \gamma \tag{10}$$

where taxation τ and public expenditure γ are expressed in terms of income.

Government expenditure can affect productivity through different channels. It can influence the capital/product ratio $1/a$ and/or the average labour productivity b:

$$a = a(\gamma) \quad with \quad a(0) > 0, \quad a' \geq 0 \quad and \quad a'' \leq 0 \tag{11}$$

$$b = b(\gamma) \quad with \quad b(0) > 0, \quad b' \geq 0 \quad and \quad b'' \leq 0 \tag{12}$$

The dynamics of the system is generated by the variations in the degree of capital utilisation in the face of discrepancies between demand and supply. If in one period the economy is not in equilibrium, in the following period the degree of capacity utilization changes:

$$u_{+1} = \psi(u) = u + \theta(g - s) \tag{13}$$

where 'x_{+1}' denotes the one-period forwarded value of the variable x and where $\theta > 0$ is the speed at which capacity utilization adjusts to the discrepancy between saving and investment.

Imposing in each period the condition $g = s$, and considering that $r \equiv \pi a u$, the solutions for u and g (obtained from equations (5),(6) and (10)) correspond to:[2]

$$u^* = \frac{\alpha + \phi(u^*)}{s_\pi \pi a(1-\gamma)} \qquad\qquad g^* = s_\pi \pi a(1-\gamma) u^* \tag{14}$$

where 'x^*' denotes the equilibrium value of the variable x. Given the shape of the investment and of the saving functions (see below Figures 1 and 7), depending on parameter values, there could be multiple equilibria.

These equilibria corresponds to the fixed points of the difference equation or 'map' (in the language of dynamical systems theory) $u = \psi(u)$. A fixed point u^* satisfies the condition $u^* = \psi(u^*)$.

An equilibrium is locally asymptotically stable or attracting, if and only if $-1 < \psi'(u^*) < 1$. From (5) , (6) and (13), this condition corresponds to

$$0 < \theta\left[s_\pi \pi a(1-\gamma) - \phi'(u^*)\right] < 2 \tag{15}$$

Assuming provisionally that $\theta < \theta^F \equiv 2\left[s_\pi \pi a(1-\gamma) - \phi'(u^*)\right]^{-1}$, corresponding to the second inequality in expression (15), an equilibrium is stable when the following condition holds:

$$s_\pi \pi a(1-\gamma) > \phi'(u^*) \tag{16}$$

[2] We assume $\alpha + \phi(0) > 0$, that is the long run expected growth of demand α is always high enough to induce positive investments even in correspondence of a low capacity utilisation in the current period (this assumption is standard in the Kaleckian literature: see Lavoie, 1996; see also Kaldor, 1940). It follows $u^* > 0$ and $g^* > 0$.

That is, for local stability at the equilibrium the slope of the saving function should be larger than the slope of the investment function.

3. The Kaleckian interpretation

3.1. Equilibrium

In the Kaleckian interpretation α represents the expected rate of growth of demand. Assuming that expectations are driven by entrepreneurs' (exogenous) animal spirits, substituting equation (8) into equations (14), we can write:

$$u^* = \frac{\bar{\alpha} + \phi(u^*)}{s_\pi \pi a(1-\gamma)} \qquad\qquad g^* = s_\pi \pi a(1-\gamma)u^* \tag{17}$$

As shown in Figure 1, plotted for (a) $\gamma = 0$, (b) $\gamma = 0.12$ and (c) $\gamma = 0.24$, there could be one, two or three equilibria.[3] In Figure 1(a) one equilibrium exists denoted by $e_L \equiv (u_L^*, g_L^*)$. In correspondence of e_L condition (16) holds, that is, the slope of the saving function is larger than the slope of the investment function. Provided that θ is small enough, e_L is globally stable. As shown in Figure 1(b), after a suitable change in parameter values other two distinct equilibria emerge (via a so-called fold or tangent bifurcation): $e_I \equiv (u_I^*, g_I^*)$ and $e_H \equiv (u_H^*, g_H^*)$, with $u_L^* < u_I^* < u_H^*$ and $g_L^* < g_I^* < g_H^*$ When three equilibria exist the intermediate equilibrium u_I^* is always locally unstable or repelling whereas, for θ sufficiently small, the other two could represent local attractors of the system. The convergence to the 'low' e_L or to the 'high' e_H equilibrium depends on the initial condition. In particular, if the economy starts within the interval $u_I^* < u_0 \le 1$, it reaches sooner or later the high equilibrium. Otherwise, if the economy starts within the interval $0 < u_0 < u_I^*$, it converges eventually to the low equilibrium. In Figure 1(c), with a further change in parameter values two equilibria disappear (via a reversed fold bifurcation). Only the high equilibrium e_H exists that, for a small θ, is globally asymptotically stable.

[3] To plot Figure 1, for the other parameters, we set the following values: $\tilde{u} = 0.5$, $\alpha = 0.07$, $a = 0.5$, $\pi = 0.4$, $s_\pi = 0.8$. As an explicit form for the nonlinear component of the investment function we choose $\phi(u) = \beta_1 \arctan(\beta_2(u - \tilde{u}))$, where $\beta_1 = 0.015$ and $\beta_2 = 15$.

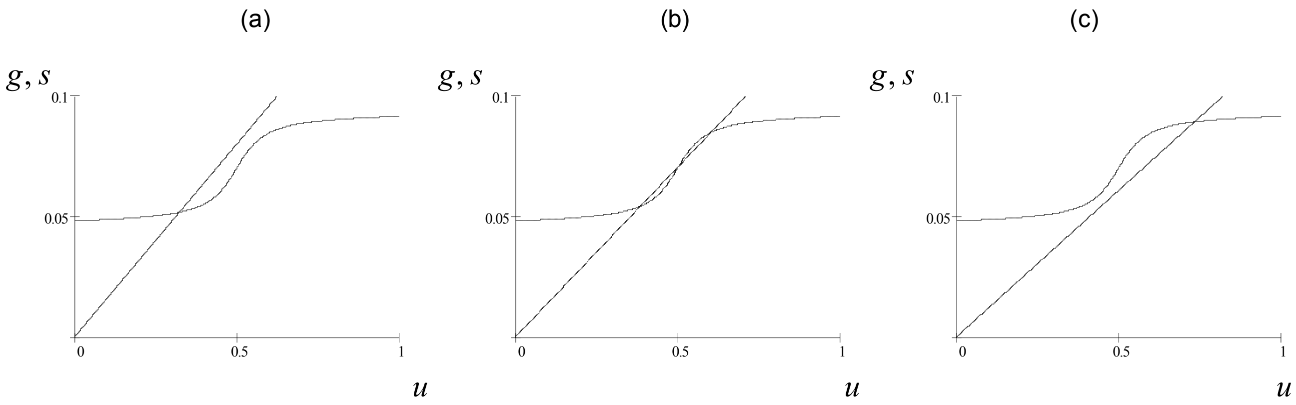

Figure 1

As shown in Figures 1(a) and 1(b), depending on parameter values, the economy could find itself trapped in a low growth equilibrium. In what follows, we suggest as possible way to escape a 'low growth trap' the implementation of a suitable public policy that could bring the economy to the condition represented in Figure 1(c).

3.2 Public expenditure effects on equilibrium capacity utilization and growth

In this section we study the effects of public expenditure on equilibrium capacity utilization and growth. To allow the comparison of steady growth equilibria, we assume that for each of them the condition $\theta < \theta^F$ holds.

3.2.1 The pure public expenditure effect

We start with the simplest hypothesis according to which public expenditure does not affect the production coefficients.

When $a'(\gamma) = 0$ and $b'(\gamma) = 0$, the effects of public expenditure on the equilibrium capacity utilisation and rate of growth can be summarised as follows:

$$\frac{du^*}{d\gamma} = \frac{s_\pi \pi a u^*}{s_\pi \pi a(1-\gamma) - \phi'(u^*)} \tag{18}$$

$$\frac{dg^*}{d\gamma} = \frac{s_\pi \pi a u^* \phi'(u^*)}{s_\pi \pi a(1-\gamma) - \phi'(u^*)} \tag{19}$$

The numerators of both derivatives are positive. It follows:

$$\frac{du^*}{d\gamma} > 0 \quad and \quad \frac{dg^*}{d\gamma} > 0 \quad for \quad s_\pi \pi a(1-\gamma) > \phi'(u^*) \text{ and}$$

$$\frac{du^*}{d\gamma} < 0 \quad and \quad \frac{dg^*}{d\gamma} < 0 \quad for \quad s_\pi \pi a(1-\gamma) < \phi'(u^*)$$

For the case of a unique equilibrium as represented in Figures 1(a) and 1(c) an increase in public expenditure increases the degree of capacity utilisation and the rate of growth. For the case of three equilibria as represented in Figure 1(b): the interior equilibrium decreases as public expenditure increases and both external equilibria increase as γ is increased. We call the effect of public expenditure on the equilibrium in the absence of any effect on the production coefficients 'pure public expenditure effect'.

Figure 2 present curves (equilibrium loci) describing the behaviour of the equilibria as γ is varied within the interval $0 \leq \gamma \leq 0.24$. As a starting point we choose the same parameters configuration as in Figure 1(a). The economy starts from an equilibrium $e_L \equiv \left(u_L^*, g_L^*\right)$ characterised by a low degree of capacity utilisation (Figure 2(a)) and a low rate of growth (Figure 2 (b)). As γ is increased, u_L^* and g_L^* increases as well. At $\gamma = \gamma^{T1}$ the system undergoes a fold bifurcation: the equilibria $e_I \equiv \left(u_I^*, g_I^*\right)$ and $e_H \equiv \left(u_H^*, g_H^*\right)$ emerge characterised respectively by intermediate and high values of capacity utilization and growth. As it is possible to read off the diagram, the value $\gamma = 0.12$ within the interval $\gamma^{T1} < \gamma < \gamma^{T2}$ corresponds to Figure 1(b). Increasing public expenditures within the interval γ^{T1} to γ^{T2} involves for the intermediate equilibrium a decrease of the degree of capacity utilization and of the rate of growth and for the external equilibria an increase of the degree of capacity utilisation and of the rate of growth. At $\gamma = \gamma^{T2}$ the system undergoes another fold bifurcation with the disappearance of the central and low equilibria. For $\gamma > \gamma^{T2}$ the high equilibrium is positively affected by the public expenditure.

The pure public expenditure effect operates as follows: the taxation necessary to finance public expenditure reduces after-tax profits and redistributes income from the private sector to the public sector which has a unitary propensity to consume. For the low and the high equilibrium, the effect on the degree of capacity utilization of such a reduction is positive. This mechanism corresponds to the so-called paradox of costs, a typical result of the Kaleckian linear model (see Rowthorn 1981;

Dutt, 1984; and Lavoie, 1995). For the intermediate equilibrium, instead, the paradox of costs does not hold.[4] The rate of growth and the degree of capacity utilisation follow the behaviour of the after-tax profits.

Finally, we note that due to the shape of the equilibrium loci, corresponding to those studied in the fold catastrophe mathematical model, an hysteresis effect emerges: starting from the low equilibrium and increasing temporarily γ, the economy moves along the lower branch. As γ passes through γ^{T2} (a fold point) a structural break occurs. The economy shift to the high equilibrium escaping the low growth trap. As γ is decreased thereafter the economy moves along the upper branch and the economy still enjoys a high growth rate as long as γ is not reduced below γ^{T1} (another fold point) after which the economy plunges again into a low growth trap.

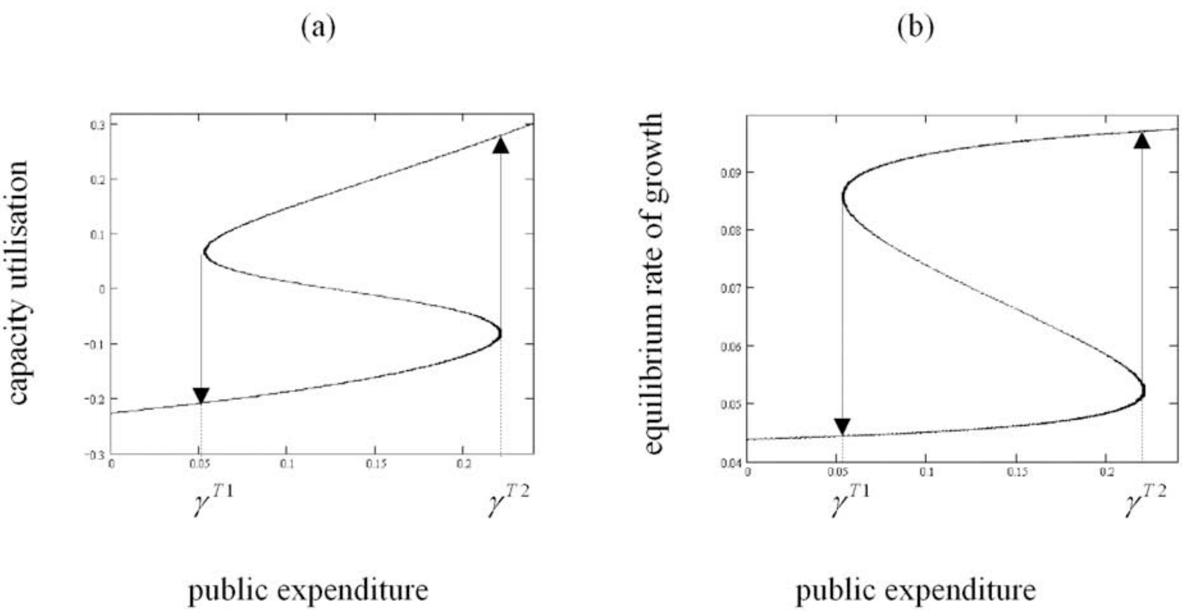

Figure 2

3.2.2 Public expenditure affects average productivity

We extend our analysis concerning the effects of public expenditure on growth by introducing a positive effect of public expenditure on the labour coefficient of production. We assume:[5]

[4] For a given degree of capacity utilization, characterizing a steady growth equilibrium, a reduction in after-tax profits implies that saving is smaller than investment. Since for the external equilibria condition (16) holds (that is, the sensitivity of saving to a change in u is greater than the sensitivity of investment), it follows that in order to equilibrate saving and investment the degree of capacity utilization should be higher. For the interior equilibrium, instead, condition (16) does not hold, therefore, to equilibrate saving and investment the degree of capacity utilization should be lower.

$$b = b(\gamma) \quad with \quad b(0) > 0, \quad b' > 0 \quad and \quad b'' \leq 0$$

Note that the change in labour productivity may or may not induce a change in the wage share depending on the relative bargaining power of unions and firms. Using equations (3), (4) and (12), we express the profit share as a function of public expenditure:

$$\pi = \pi(\gamma) = 1 - w_0 \left[b(\gamma)\right]^{\lambda - 1} \tag{20}$$

We have that

$$\pi'(\gamma) = w(1-\lambda)b^{\lambda-2}b'(\gamma) \geq (<)\, 0 \quad for \quad \lambda \leq (>) 1$$

That is, if the wage rate increase less (more) than labour productivity, the profit share increases (decreases). Note also that letting $\lambda < 1$, it follows that $\pi' > 0$ and $\pi'' < 0$

The degree of capacity utilisation and the rate of capital accumulation are affected by changes in γ (via improvements of average labour productivity) as follows:

$$\frac{du^*}{d\gamma} = \frac{s_\pi a[\pi - \pi'(\gamma)(1-\gamma)]u^*}{s_\pi \pi a(1-\gamma) - \phi'(u^*)} = \frac{s_\pi \pi a u^*}{s_\pi \pi a(1-\gamma) - \phi'(u^*)} - \frac{s_\pi a \pi'(\gamma)(1-\gamma)u^*}{s_\pi \pi a(1-\gamma) - \phi'(u^*)} \tag{21}$$

$$\frac{dg^*}{d\gamma} = \frac{s_\pi a[\pi - \pi'(1-\gamma)]\phi'(u^*)u^*}{s_\pi \pi a(1-\gamma) - \phi'(u^*)} \tag{22}$$

From a comparison between the derivatives (18)-(19) and (21)-(22), there are two effects that public expenditure exerts on the equilibrium: the 'pure public expenditure effect', due to the reduction in after-tax profits caused by the increase in taxation, as in the previous case, and the 'productivity effect' following the change in distribution between wages and profits, induced by the change in productivity.

The sign of the derivatives (21) and (22) depends on the impact of public expenditure on after-tax profits, $\left[\pi(1-\gamma)\right]' = \pi'(\gamma)(1-\gamma) - \pi$.

If the inequality

[5] We do not present here the case according to which public expenditure affects the capital input coefficient a. The analysis concerning this case is qualitatively identical to the one we develop for the case $b'(\gamma) > 0$ and $\pi'(\gamma) > 0$.

$$\pi'(\gamma) < \frac{\pi(\gamma)}{1-\gamma} \tag{23}$$

holds, after-tax profits fall and the degree of capacity utilisation and the rate of growth rise; whereas, if the reversed inequality

$$\pi'(\gamma) > \frac{\pi(\gamma)}{1-\gamma} \tag{24}$$

holds, after-tax profits increase and the degree of capacity utilisation and the rate of growth decrease.

Condition (23) implies one of this two cases: (i) $\pi'(\gamma)$ is negative. The effects of public expenditure on labour productivity, which translate into a less than proportional increase in the profit share, acts in the same direction of an increase in the tax rate on profits; (ii) $\pi'(\gamma)$ is positive but the effects of public expenditure on the profit share are not strong enough to countervail those of an increase in taxation. The overall impact on after-tax profits is negative. Conversely, condition (24) means that the productivity effect is stronger than the demand effect. The impact on after-tax profits is necessarily positive.

When $\pi'(\gamma) \leq 0\ (\lambda \geq 1)$ or $0 < \pi'(0) < \pi(0)$ – given that $\pi''(\gamma) < 0$ for $\lambda < 1$ – condition (23) holds for any γ. If condition (23) is always satisfied, the analysis of the effects of public expenditure on u^* and g^* is similar to that shown in Figure 2. This is because, the change in distribution between wages and profits induced by productivity improvements reinforces (or is never strong enough to countervail) the pure public expenditure effect.

Assuming $\pi(0) < \pi'(0) < \infty$, we have that condition (23) holds for $\gamma > \bar{\gamma}$, whereas condition (24) holds for $0 \leq \gamma < \bar{\gamma}$, where $\bar{\gamma}$ solves $\pi'(\bar{\gamma}) = \frac{\pi(\bar{\gamma})}{1-\bar{\gamma}}$.

Considering the external equilibria $\left(u_L^*, g_L^*\right)$ and $\left(u_H^*, g_H^*\right)$, for which condition (16) holds, we have that

$\frac{du^*}{d\gamma} < 0$ and $\frac{dg^*}{d\gamma} < 0$ for $\gamma < \bar{\gamma}$ and

$$\frac{du^*}{d\gamma} > 0 \quad \text{and} \quad \frac{dg^*}{d\gamma} > 0 \text{ for } \gamma > \bar{\gamma}$$

In words, when $\gamma < \bar{\gamma}$ the productivity effect is stronger than the pure public expenditure effect so that an increase in public expenditure rises after-tax profits, following a negative impact on the equilibrium degree of capacity utilization and on the equilibrium rate of growth. On the contrary, when $\gamma > \bar{\gamma}$ the pure public expenditure effect is sufficiently strong to counteract the productivity effect, so that after-tax profits decline: equilibrium capacity utilization and growth increase. Consequently, the condition that maximizes after-tax profits $\gamma = \bar{\gamma}$, minimizes the degree of capacity utilisation and the growth rate, provided that condition (16) holds (see Figure 4).

For the internal equilibrium e_I, for which the reversed inequality (16) holds, the opposite occurs. That is, for $\gamma < \bar{\gamma}$ the productivity effect is weaker than the pure public expenditure effect and public expenditure has a positive impact on capacity utilization and growth. Instead, for $\gamma > \bar{\gamma}$ the productivity effect is stronger than the pure public expenditure effect. Equilibrium capacity utilization and growth decrease when public expenditure increases. Consequently, after-tax profits are at their minimum at $\gamma = \bar{\gamma}$ and the internal equilibrium, if it exists, could have a maximum (see Figure 4).

Typically, in the standard linear Kaleckian model, the phenomenon known as the ‘paradox of costs’ arises from a reduction in the profit share /an increase in the wage share (via labour productivity improvements), which induces – at the equilibrium – an increase in the degree of capacity utilization and in the rate of growth. In our model, the paradox of costs is a possible consequence of an increase in the government size, from which follow a reduction (increase) in after-tax profits and an increase (reduction) in the degree of capacity utilisation and in the rate of growth. It occurs when condition (16) is verified.

Figure 3 present surfaces or equilibrium loci which describe the behaviour of the equilibria for the case $b'(\gamma) > 0$ varying public expenditure within the interval $0 \le \gamma \le 0.5$ and the wage-productivity elasticity λ within $0.6 \le \lambda \le 0.85$ – more specifically, we increase its complement to 1, $(1-\lambda)$, from 0.15 to 0.4.

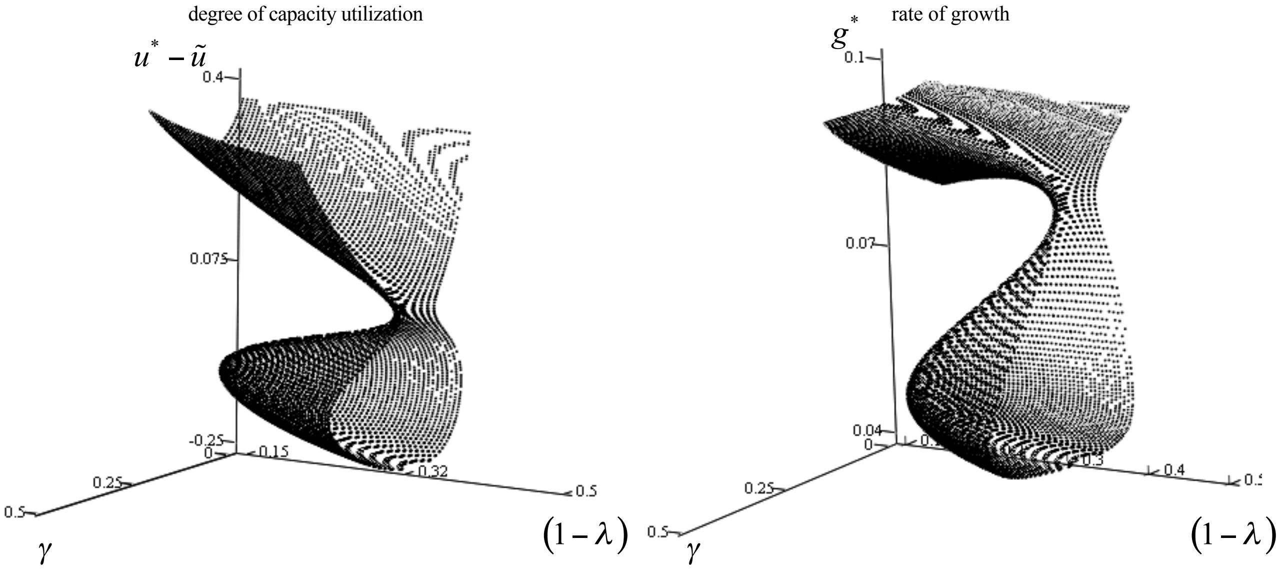

Figure 3

In Figure 4, we study in detail three cases with decreasing wage-productivity elasticity which corresponds to specific sections of the surface plots in Figure 3:[6] in panel (a) π is barely affected by changes in γ: $\lambda = 0.83$ $(1-\lambda = 0.17)$; in panel (b) it is quite responsive to changes in γ: $\lambda = 0.628$ $(1-\lambda = 0.372)$; finally, in panel (c) it has an even higher degree of sensitiveness to changes in γ: $\lambda = 0.6$ $(1-\lambda = 0.4)$. As a starting point, we choose a parameters configuration which has a crucial difference compared to the one we have used to plot Figure 2. That is, we have chosen $\pi(0)$ such that in Figures 3 and 4, the economy starts from the high equilibrium similarly to the case presented in Figure 1(c). This allows us to study all the possible behaviours that the equilibrium loci can undertake as public expenditure varies.

In Figure 4(a) only the high equilibrium exists which decreases as γ increases until $\gamma = \overline{\gamma}$ and increases after that.

Reducing λ strengthens the pure public expenditure effect generating more complex phenomena. In Figure 4(b) at $\gamma = \gamma^T$ the system undergoes a fold bifurcation: the equilibria e_I and e_L emerge. For $\gamma < \overline{\gamma}$ increasing public expenditure reduces the high and the low equilibrium, whereas the

[6] In our simulations, we assumed that $b(\gamma) = b_0 + b_1 \arctan(b_2 \gamma)$. The parameters constellation chosen corresponds to $b_0 = 1$, $b_1 = 1.15$, $b_2 = 7.5$, $w_0 = 0.72$, $b_0 = 1$.

intermediate equilibrium increases. Conversely, for $\gamma > \bar{\gamma}$ the external equilibria increase with γ, whereas the intermediate equilibrium decreases. $\bar{\gamma}$ represents a minimum for e_L and e_H and a maximum for e_I. At $\gamma = \gamma^{T2}$ the system undergoes a reversed fold bifurcation with the disappearance of the low and the intermediate equilibria. The hysteresis effect is unidirectional from the low equilibrium to the high equilibrium: reducing γ below $\gamma = \gamma^{T1}$ or increasing it above $\gamma = \gamma^{T2}$ allow the system to shift permanently from the low to the high equilibrium.

Finally, in Figure 4(c) the equibrium curve is folded twice and four fold point appears: γ^{T1} and γ^{T2} located at the left of $\bar{\gamma}$ and γ^{T3} and γ^{T4} located at the right of $\bar{\gamma}$. Following these complicated curves the system could undertake one ore even two catastrophic (structural) changes. Indeed, starting from an high equilibrium at $\gamma = 0$, the system could plunge into a low equilibrium as it crosses γ^{T2} and then it could shift back to a high equilibrium crossing γ^{T4}. Therefore, due to the strong productivity effect, for low values of γ increasing public expenditure could worsen dramatically the growth perspectives of the economy. This result does not arise if the system starts from a low equilibrium.

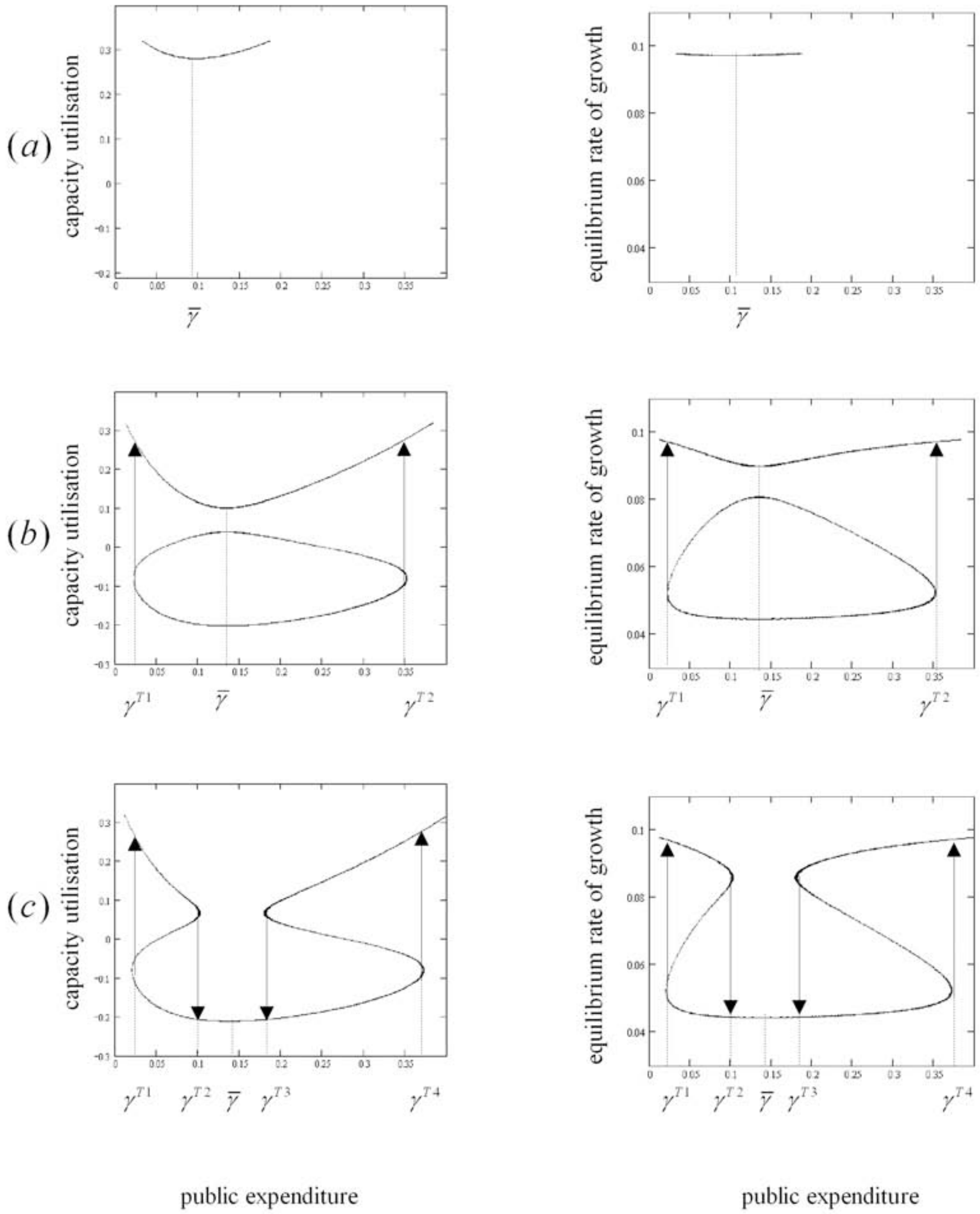

Figure 4

3.3 Dynamic analysis

In this section, we let vary the parameter θ, which represents the speed at which capacity utilization adjusts to excess demand. We first study how the long term behaviour of the economy changes when this parameter is increased. We also explore how public expenditure impacts on such behaviour.

The central equation of the dynamic analysis is the difference equation or map (13) that, for the Kaleckian case, we explicit as follows:

$$\psi(u) = u + \theta\left[\alpha + \phi(u) - s_\pi \pi a(1-\gamma)u\right]$$

We notice that θ has no effect on the fixed point solution $\psi(u^*) = u^*$. This is confirmed by the fact that this parameter has no bearing on condition (16), which, as we discussed above, determines if an equilibrium exists and how many equilibria there are. Changing θ, however, affects the local stability properties of fixed points and the global stability properties of the map $\psi(u)$. In particular, concerning a fixed point u^*, a necessary and sufficient condition for local stability is

$$0 < \theta < \theta^F \equiv \frac{2}{s_\pi \pi a(1-\gamma) - \phi'(u^*)}$$

The fixed point u^* is attracting for $0 < \theta < \theta^F$; when θ crosses θ^F a Flip or period doubling bifurcation occurs. u^* loses stability and a period two cycle emerges which is locally stable. That is, the long period behaviour of the economic system corresponds to the continual repetition of two values of the degree of capacity utilisation (and of the other time varying variables of the model). Increasing further θ many period doubling bifurcation occurs, regular cycles of any order and irregular cycles emerge (see Figure A.2 in the Appendix).

In the Appendix, we show that the period doubling sequence to complex behaviour could occur both for the low and the high equilibrium, u_L^* and u_H^*, whereas for the intermediate equilibrium u_I^* a Flip bifurcation never occurs – we denote by θ_L^F and θ_H^F the bifurcation values of θ corresponding to the low and to the high equilibrium. Moreover, we show there that also the global stability properties of the map increase in complexity as θ is increased, depending on the level of public expenditure γ. These phenomena are typical to bimodal maps (i.e., maps characterised by two extrema), a class of maps to which $\psi(u)$ belongs for values of θ sufficiently high.

Moreover, we describe in analytical detail what follows: in the first place, for a value of γ which gives rise to three equilibria, as θ is increased, two coexisting attractors of various periodicity and chaotic attractors emerge: the first attractor cycling around the low equilibrium and the second cycling around the high equilibrium. The two attractors are asymmetric to each other and, most of the times, they do not have the same periodicity. In the second place, defining a basin of attraction as the set of initial values that converge to an attractor, the structure of the basins of attraction of the two attractors undergoes substantial modifications. Whereas for θ small the structure of the two basins of attraction is simple, corresponding respectively to the interval at the left and to the interval at the right of the intermediate equilibrium u_I^*, increasing θ they become more and more disconnected and intermingled. From the point of view of a policy maker, the effect of a change in public expenditure becomes more and more difficult to predict, since hysteresis phenomena may emerge. Indeed, even a small change in γ could induce a shift of the economy from the one attractor (for example, the one cycling around the high equilibrium) to the other (for example, the one cycling around the low equilibrium). For θ sufficiently large, a so-called global bifurcation takes place: one attractor disappears and almost all initial conditions converge to the other attractor. The existence of a global bifurcation point could result in an unexpected catastrophic consequence of a change in public expenditure. In the appendix, we present a simulation study in which, following a small increase in γ, an economy, which was settled on a chaotic attractor cycling around the high equilibrium u_H^*, plunges into a period two-cycle cycling around the low equilibrium u_H^*, worsening notably the economic conditions.

We now explore, albeit briefly, the impact of a change in γ on the dynamic properties of the map $\psi(u)$. Concerning the effect on the local stability properties of the fixed points. For the case in which public expenditure does not affect the production coefficients, we have that

$$\frac{d\theta^F}{d\gamma} = \theta^F \frac{s_\pi \pi a + \phi''(u^*)\, du^*/d\gamma}{s_\pi \pi a(1-\gamma) - \phi'(u^*)}.$$

Since $\phi''(u_L^*) > 0$ and $\frac{du^*}{d\gamma} > 0$, it follows $\frac{d\theta_L^F}{d\gamma} > 0$, whereas since $\phi''(u_H^*) < 0$ the sign of $\frac{d\theta_H^F}{d\gamma}$ is indeterminate. That is, public expenditure has a stabilising effect on the low equilibrium since it increases the value of θ at which a Flip bifurcation takes place. The effect on the high equilibrium, however, is not univocal.

Figure 5, which deals with the case in which public expenditure does not affect the production coefficients, presents bifurcation diagram showing the impact of $0 \le \gamma \le 0.4$ on the long term behaviour of the degree of capacity utilisation for (a) $\theta = 10$, (b) $\theta = 15$ and (c) $\theta = 20$.[7] In Figure 5(a), the low equilibrium is stable for $0 \le \gamma < \gamma^T$. As γ crosses γ^T a fold bifurcation takes place, the low and the intermediate equilibrium (where the latter equilibrium is not visible in the diagram) disappears and the high equilibrium emerges, which is attracting for $\gamma^T < \gamma \le 0.4$. In Figure 5(b) a Flip bifurcation γ_L^F is visible, occurring for the low equilibrium. In particular, for $0 \le \gamma < \gamma_L^F$, the

[7] To plot Figure 5 we used, for the other parameters, the same values as in Figure 2 and we set as initial condition $u_0 = 0.51$.

low equilibrium is locally unstable or repelling and an attracting period two cycle is visible in the diagram, where γ_L^F solves $\psi'(u_L^*) = -1$ and it is analogous to θ_L^F. Increasing *γ*, the low equilibrium gains stability at γ_L^F. In Figure 5(c) two Flip bifurcations are visible for the high equilibrium: γ_H^{F1} and γ_H^{F2}, which solve $\psi'(u_H^*) = -1$. u_H^* is stable for $\gamma^T < \gamma < \gamma_H^{F1}$. At γ_H^{F1} the high equilibrium loses local stability and an attracting period two cycle emerges. Increasing further public expenditure as *γ* crosses γ_H^{F2}, the period two cycle disappears and the high equilibrium becomes again attracting for $\gamma_H^{F2} < \gamma \le 0.4$. Concerning the low equilibrium, it is always repelling for $0 \le \gamma < \gamma^T$. However, to larger values of *γ* corresponds less wide fluctuations.

Figure 5 confirms that public expenditure has a stabilising effect on the low equilibrium and also on the global dynamics, reducing the amplitude of the economic fluctuations. Moreover, the simulation shows that *γ* has at first a destabilising effect and after a stabilising effect on the high equilibrium.

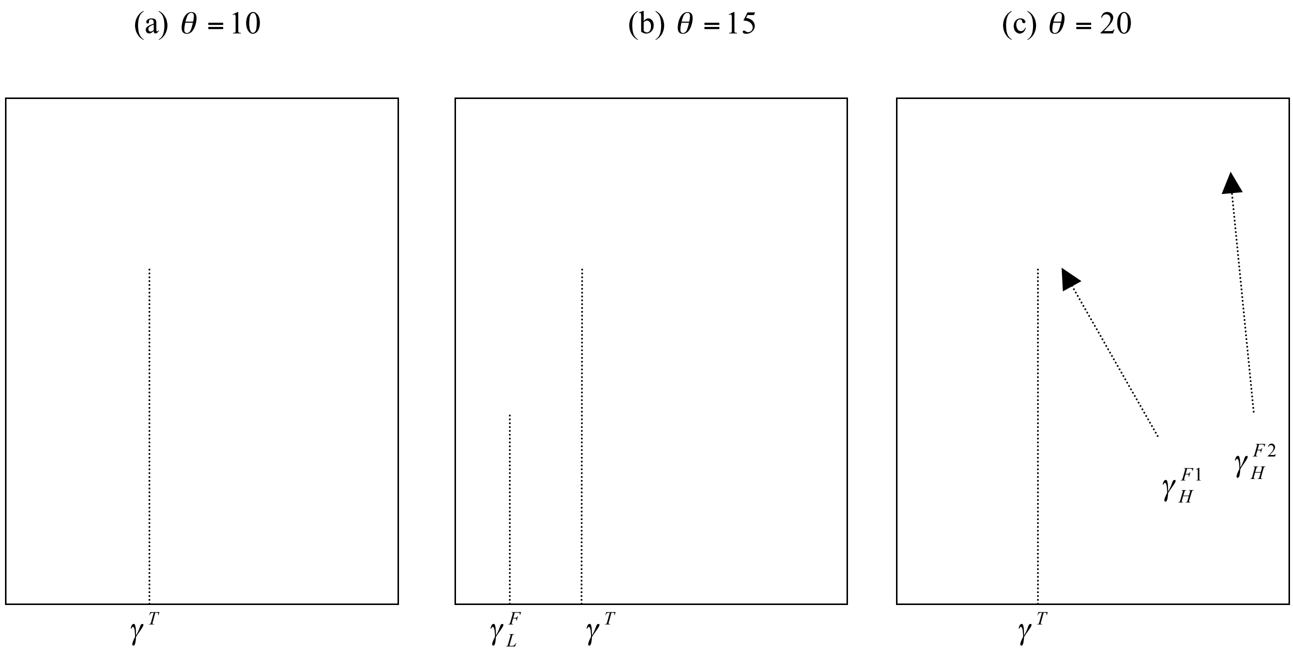

Figure 5

For the case in which public expenditure affects labour productivity, we have that

$$\frac{d\theta^F}{d\gamma} \equiv \theta^F \frac{s_\pi a\left[\pi(\gamma) - \pi'(\gamma)(1-\gamma)\right] + \phi''(u^*)\dfrac{du^*}{d\gamma}}{s_\pi \pi(\gamma) a(1-\gamma) - \phi'(u^*)}$$

When condition (23) holds, i.e. $\pi - \pi'(\gamma)(1-\gamma) > 0$, since $\phi''(u_L^*) > 0$ and $\frac{du^*}{d\gamma} > 0$, it follows $\frac{d\theta_L^F}{d\gamma} > 0$, whereas since $\phi''(u_H^*) < 0$ the sign of $\frac{d\theta_H^F}{d\gamma}$ is indeterminate. That is, when $\gamma > \overline{\gamma}$, public expenditure has a stabilising effect on the low equilibrium since it increases the value of θ at which a Flip bifurcation takes place. The effect on the high equilibrium, however, is not univocal; on the other hand, when condition (24) holds, i.e. $\pi - \pi'(\gamma)(1-\gamma) < 0$, since $\phi''(u_L^*) > 0$ and $\frac{du^*}{d\gamma} < 0$, it follows $\frac{d\theta_L^F}{d\gamma} < 0$; whereas the sign of $\frac{d\theta_H^F}{d\gamma}$ is indeterminate. That is, when $\gamma < \overline{\gamma}$, public expenditure has a destabilising effect on the low equilibrium. The effect on the high equilibrium is not univocal.

Figure 6, which deals with the case in which public expenditure affects labour productivity, presents bifurcation diagrams showing the impact of $0 \leq \gamma \leq 0.4$ on the long term behaviour of the degree of capacity utilisation for (a) $\theta = 15$, (b) $\theta = 17.5$ and (c) $\theta = 20$.[8] In Figure 6(a), the high equilibrium is stable for $0 \leq \gamma < \gamma^{T1}$ and $\gamma^{T2} < \gamma \leq 0.4$ and the low equilibrium for $\gamma^{T1} < \gamma < \gamma^{T2}$. At $\gamma = \gamma^{T1}$ a fold bifurcation occurs, the low and the intermediate equilibrium (where the intermediate equilibrium is not visible in the diagram) emerge; whereas at the fold bifurcation $\gamma = \gamma^{T2}$, the low and the intermediate equilibrium disappear. In Figure 6(b), two Flip bifurcations are visible for the low equilibrium γ_L^{F1} and γ_L^{F2}. The first occurs on the left of $\overline{\gamma}$ and the second on the right. u_L^* is stable for $\gamma^{T1} < \gamma < \gamma_L^{F1}$ and for $\gamma_L^{F2} < \gamma < \gamma^{T2}$. At γ_L^{F1} the low equilibrium loses stability and an attracting period two cycle emerges, whose amplitude increases until $\gamma = \overline{\gamma}$ and decreases thereafter. At $\gamma = \gamma_L^{F2}$ the period two cycle disappears and the low equilibrium becomes attracting again. In Figure 6(c), two Flip bifurcations are visible for the high equilibrium, γ_H^{F1} and γ_H^{F2}, located at the left and at the right of $\overline{\gamma}$, respectively. Looking at the left of $\overline{\gamma}$, for $0 \leq \gamma < \gamma_H^{F1}$, the high equilibrium is repelling and an attracting period two cycle is visible. Increasing γ, the high equilibrium gains stability at γ_H^{F1}. Looking at the right of $\overline{\gamma}$, the opposite occurs: u_H^* loses stability at γ_H^{F2}. Increasing γ, an attractive period two cycle emerges, whose amplitude is widening. Concerning the low equilibrium, it is always repelling for $\gamma^{T1} < \gamma < \gamma^{T2}$. We can notice, however,

[8] To plot Figure 6 we used, for the other parameters, the same values as in Figure 4(c) and we set as initial condition $u_0 = 0.51$.

that fluctuations are wider for values of public expenditure close to $\bar{\gamma}$ and narrower for values of γ far from $\bar{\gamma}$.

Figure 6 confirms that when $\gamma < \bar{\gamma}$, public expenditure has a destabilising effect on the low equilibrium and a stabilising effect on the high equilibrium. On the contrary, when $\gamma > \bar{\gamma}$, public expenditure has a stabilising effect on the low equilibrium and a destabilising effect on the high equilibrium. The effect on the global dynamic is, instead, more ambiguous. Increasing public expenditure could both increase or decrease the amplitude of economic fluctuations.

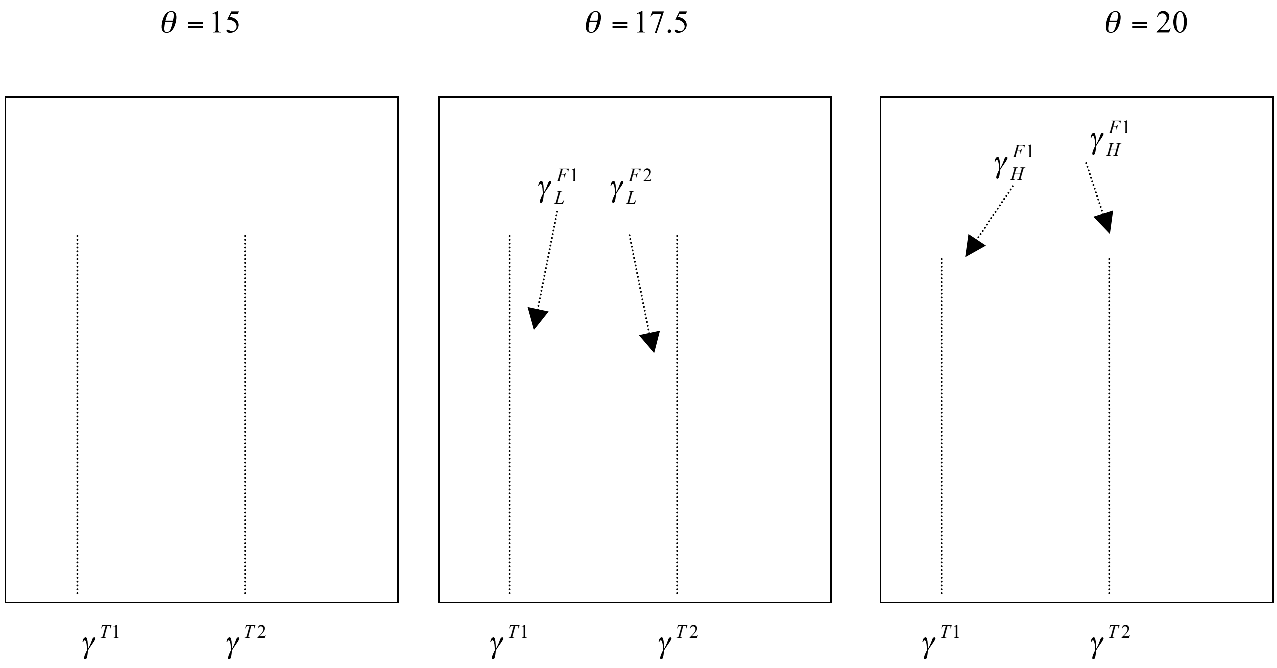

Figure 6

4. The Classical-Harrodian interpretation

4.1 Equilibrium

In the case in which the expected rate of growth of demand is equal to the Harrodian warranted rate of growth, that is, $\alpha = \tilde{g} \equiv s_\pi \pi a(1-\gamma)\tilde{u}$, the solutions for u and g become

$$u^* = \frac{\tilde{g} + \phi(u^*)}{s_\pi \pi a(1-\gamma)} \quad \text{and} \quad g^* = \tilde{g} + \phi(u^*)$$

As shown in Figure 7, depending on parameter values, there could be one or three equilibria.[9] In Figure 7(a) only the equilibrium corresponding to the warranted rate of growth exists, denoted by $\tilde{e} \equiv (\tilde{u}, \tilde{g})$. In correspondence of $\tilde{e}$ condition (15) holds, i.e., the slope of the saving function is larger than the slope of the investment function. Provided that θ is small enough, $\tilde{e}$ is globally stable. As shown in Figure 7(b) after a suitable change in parameter values, two distinct equilibria emerge (via a so-called pitchfork bifurcation): $e_L \equiv (u_L^*, g_L^*)$ and $e_H \equiv (u_H^*, g_H^*)$, which lie symmetrically around $\tilde{e}$. Symmetry implies $\phi(u_L^*) = -\phi(u_H^*)$, it follows $u_L^* < \tilde{u} < u_H^*$ and $g_L^* < \tilde{g} < g_H^*$. When three equilibria exist the equilibrium $\tilde{e}$ is always unstable, whereas, for a θ sufficiently small, the other two – for which condition (15) holds – could represent local attractors of the system. The convergence to the 'low' equilibrium e_L or to the 'high' equilibrium e_H depends on the initial condition. In particular, if the economy starts within the interval $\tilde{u} < u_0 \leq 1$, it reaches eventually the high equilibrium. Conversely, if the economy starts within the interval $0 < u_0 < \tilde{u}$, it converges sooner or later to the low equilibrium.

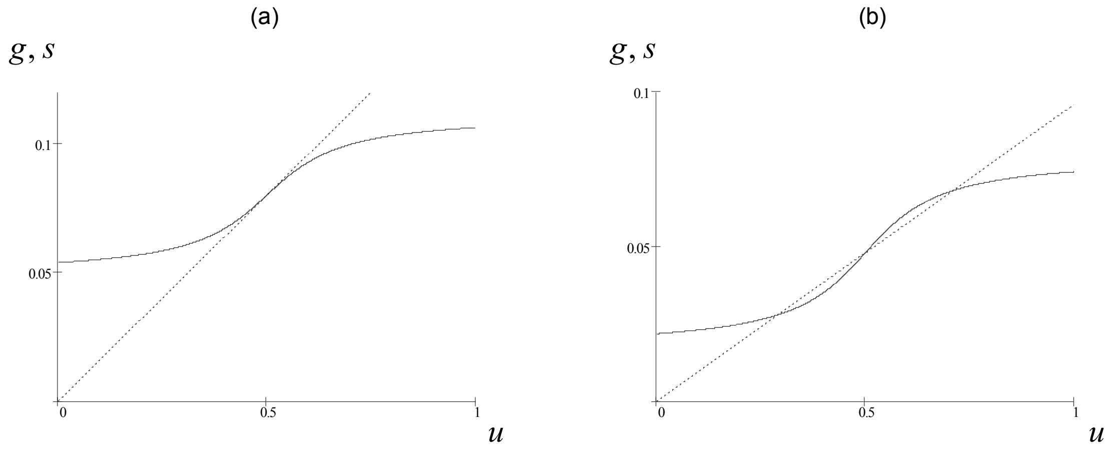

Figure 7

A crucial difference from the Kaleckian case is that, due to the symmetry of the two external equilibria – they emerge or disappear simultaneously –, no hysteresis phenomena arise. Once the it is locked in the low equilibrium an expansive public policy is less effective in releasing the economy from it.

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[9] To plot Figure 7, we used the following parameters constellation: $\tilde{u} = 0.5$, $a = 0.5$, $s_\pi = 0.8$, $\alpha = 0.02$, β = 7.5, $\gamma = 0$ for panel (a) and $\gamma = 0.4$ for panel (b). Moreover we assumed that public expenditure does not affect the profit share setting for both panels $\pi = 0.4$.

4.2 Public expenditure effects on equilibrium capacity utilisation and growth

In this section, we study the effects of public expenditure on equilibrium capacity utilisation and growth for the Classical-Harrodian case. To allow the comparison of steady growth equilibria, we assume that for each of them the condition $\theta < \theta^F$ holds.

4.2.1 The pure public expenditure effect

As for the Kaleckian case, we start with the simplest hypothesis according to which public expenditure does not affect the production coefficients.

When $a'(\gamma) = 0$ and $b'(\gamma) = 0$, the effects of public expenditure on the equilibrium capacity utilisation and the rate of growth can be summarised as follows:

$$\frac{du^*}{d\gamma} = \frac{s_\pi \pi a(u^* - \tilde{u})}{s_\pi \pi a(1-\gamma) - \phi'(u^*)} \tag{25}$$

$$\frac{dg^*}{d\gamma} = \frac{d\tilde{g}}{d\gamma} + \phi'(u^*)\frac{du^*}{d\gamma} = -s_\pi \pi a \frac{s_\pi \pi a(1-\gamma)\tilde{u} - \phi'(u^*)u^*}{s_\pi \pi a(1-\gamma) - \phi'(u^*)} \tag{26}$$

Compared with equations (18) and (19), which hold for the Kaleckian case, equations (25) and (26) show that the effect of public expenditure on equilibrium involves also a change in the warranted rate of growth, that is, in the entrepreneurs' long-run expectations. If the impact of γ on the warranted rate of growth is sufficiently strong, the direction of the 'pure public expenditure effect' on the degree of capacity utilisation is reversed. The influence of public expenditure on the rate of growth is also modified.

Going into detail, for the equilibrium $\tilde{e}$, corresponding to the warranted rate of growth, public expenditure has no effect on the degree of capacity utilisation and it affects negatively the long-run rate of growth:

$$\frac{d\tilde{u}}{d\gamma} = 0 \quad \text{and} \quad \frac{d\tilde{g}}{d\gamma} = -s_\pi \pi a \tilde{u} < 0 .$$

The negative effect on the rate of growth is due to the increase in taxation, necessary to finance public expenditure, that reduces after-tax profits and saving at normal capacity utilisation.

For the external equilibria, when they exist, matters are more complicated. For the low equilibrium, the degree of capacity utilisation and the rate of growth decrease as public expenditure increase:

$$\frac{du_L^*}{d\gamma} < 0 \quad \text{and} \quad \frac{dg_L^*}{d\gamma} = -s_\pi \pi a\tilde{u} + \phi'(u_L^*)\frac{du_L^*}{d\gamma} < 0 .$$

The negative impact of public expenditure on the degree of capacity utilisation in the low equilibrium can be explained as follows: for a given degree of capacity utilisation, the reduction in the after tax profit, following the increase in taxation, implies a reduction in saving smaller than a reduction in investment: consequently, saving exceeds investment. Since for e_L condition (16) holds – that is, the sensitivity of saving to changes in u is greater than the sensitivity of investment – in order to equilibrate saving and investment, the degree of capacity utilisation should decrease. Moreover, in the low equilibrium, as public expenditure increases, the rate of growth decreases faster than the warranted rate of growth due to the negative impact on the degree of capacity utilisation.

For the high equilibrium, increasing γ the degree of capacity utilisation increases, whereas the rate of growth increases or decreases, depending on the level of public expenditure:

$$\frac{du_H^*}{d\gamma} > 0 \quad \text{and} \quad \frac{dg_H^*}{d\gamma} = -s_\pi \pi a\tilde{u} + \phi'(u_H^*)\frac{du_H^*}{d\gamma} \geq (<)\, 0 \quad \text{for} \quad \gamma \leq (>) \hat{\gamma} ,$$

where $\hat{\gamma}$ solves $s_\pi \pi a\tilde{u}(1-\hat{\gamma}) = \phi'(u_H^*)u_H^*$.

The positive impact of public expenditure on the degree of capacity utilisation in the high equilibrium can be explained as follows: for a given degree of capacity utilisation, the reduction in the after tax profit, following the increase in taxation, implies a reduction in saving greater than the reduction in investment: consequently, saving falls short of investment. Since for e_H condition (16) holds – that is, the sensitivity of saving to changes in u is greater than the sensitivity of investment – in order to equilibrate saving and investment, the degree of capacity utilisation should rise. Moreover, in the high equilibrium, due to the positive impact of public expenditure on the degree of capacity utilisation, the rate of growth increases for $\gamma < \hat{\gamma}$ and decreases (but less than the warranted rate of growth) for $\gamma > \hat{\gamma}$; it reaches its maximum at $\gamma = \hat{\gamma}$.

The condition for a positive impact of public expenditure on the equilibrium rate of capital accumulation can be expressed concisely as follows:

$$s_\pi \pi a\tilde{u}(1-\gamma) < \phi'(u^*)u^*$$

Since $\partial g^* / \partial \pi(1-\gamma) = s_\pi a\tilde{u}$ and $\partial g^* / \partial u^* = \phi'(u^*)$ this inequality can be rewritten as:

$$\eta_{\pi(1-\gamma)} \equiv \frac{\partial g^*}{\partial \pi(1-\gamma)} \frac{\pi(1-\gamma)}{g^*} < \eta_{u^*} \equiv \frac{\partial g^*}{\partial u^*} \frac{u^*}{g^*} \tag{27}$$

that is, the partial elasticity of the rate of capital accumulation with respect to the degree of capital utilisation, η_u, should be larger than the partial elasticity with respect to the after tax profit, $\eta_{\pi(1-\gamma)}$. For the low equilibrium and the equilibrium corresponding to the warranted growth path, condition (27) does not hold; whereas for the high equilibrium, it holds for $\gamma < \hat{\gamma}$ (and it is not satisfied for $\gamma > \hat{\gamma}$).

Figure 8 presents curves (equilibrium loci) describing the behaviour of the equilibria as γ is varied within the interval $0 \le \gamma \le 24$. As a starting point we choose the same parameters constellation as in Figure 7(a). The economy starts from the equilibrium $\tilde{e}$ corresponding to the warranted rate of growth, which is globally stable. Increasing γ, the normal degree of capacity utilization $\tilde{u}$ is unaffected (Figure 8(a)), whereas the warranted rate of growth $\tilde{g}$ decreases linearly (Figure 8(b)). At $\gamma = \gamma^P$ the system undergoes a pitchfork bifurcation: the locally stable equilibria e_L and e_H emerge simultaneously and the equilibrium $\tilde{e}$ becomes unstable. As shown in Figure 8(a) the corresponding values of the degree of capacity utilisation, i.e., u_L^* for the low equilibrium and u_H^* for the high equilibrium, lie symmetrically around $\tilde{u}$. u_L^* decreases and u_H^* increases with γ. As shown in Figure 8(b), increasing γ, the rate of growth corresponding to the low equilibrium g_L^* decreases faster than the warranted rate of growth; whereas the rate of growth corresponding to the high equilibrium g_H^* increases until $\gamma = \hat{\gamma}$ and decreases thereafter but a slower pace than $\tilde{g}$.

Figure 8 suggests that, as γ rises and the after-tax profit decreases, the economy could experience three alternative steady growth regimes depending on the initial condition and on the level of public expenditure. In the first regime, which is profit-led, the degree of capacity utilisation is constant or falling and the rate of growth is also falling. Such a regime occurs when the economy moves along the warranted growth path or along the lower equilibrium. In the second regime, which is also profit-led, the degree of capacity utilisation is increasing whereas the rate of growth is falling. Such a regime occurs when the economy moves along the high equilibrium and $\gamma > \hat{\gamma}$; finally, in the third regime, which is public consumption-led, both the degree of capacity utilisation and the rate of growth increase. Such a regime occurs when the economy moves along the high equilibrium and $\gamma < \hat{\gamma}$. The third regime is the only one for which the paradox of costs hold: the rate of growth and the degree of capacity utilisation both moves in the opposite direction of the after-tax profit.[10]

Finally, we note that, in contrast to the Kaleckian case, in the Classical-Harrodian case no hysteresis effect emerges comparing equilibria. If the economy starts below $\tilde{e}$, it converges sooner or later to e_L; while, if it starts above $\tilde{e}$, it converges eventually to e_H. Once the economy lies on the lower or on the upper branch of the equilibrium loci plotted in Figure 8, increasing public expenditure has only a smooth effect on the long term position of the economy. For example, if the system is locked in the low equilibrium, increasing public expenditure reduces continuously the rate of growth.

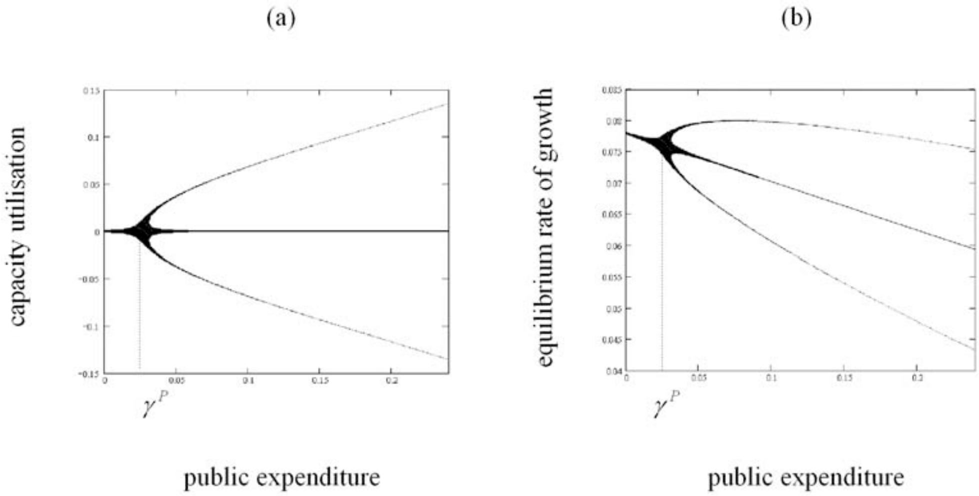


Figure 8

4.2.2 Public Expenditure affects average productivity

We extend our analysis concerning the effects of public expenditure on growth by introducing a positive effect of public expenditure on the labour coefficient of production.[11] As discussed previously, the change in labour productivity, may or may not induce a change in the wage share depending on the relative bargaining power of unions and firms. As before (see equation (20)), we express the profit share as a function of public expenditure: $\pi = \pi(\gamma)$.

[10] The third regime is analogous to the wage-led growth regime, which applies to the standard Kaleckian model with no government sector when the wage share varies.

[11] We do not present here the case according to which public expenditure affects the capital coefficient a. The analysis concerning this case is qualitatively identical to the one we develop for the case $b'(\gamma) > 0$.

The degree of capacity utilisation and the rate of capital accumulation are affected by changes in γ (via improvements of average labour productivity) as follows:

$$\frac{du^*}{d\gamma} = \frac{s_\pi a[\pi - \pi'(1-\gamma)](u^* - \tilde{u})}{s_\pi \pi a(1-\gamma) - \phi'(u^*)} \tag{28}$$

$$\frac{dg^*}{d\gamma} = \frac{d\tilde{g}}{d\gamma} + \phi'(u^*)\frac{du^*}{d\gamma} \tag{29}$$

From a comparison between the derivatives (25)-(26) and (28)-(29), two are the effects that public expenditure exerts on the equilibrium: the 'pure public expenditure effect', whose features were discussed above, due to the reduction in after-tax profits caused by the increase in taxation; and the 'productivity effect' following the change in distribution between wages and profits, induced by the change in productivity.

The direction of the productivity effect depends on the impact of public expenditure on after-tax profits, $[\pi(\gamma)(1-\gamma)]' = \pi'(\gamma)(1-\gamma) - \pi$. As discussed above, when $\pi'(\gamma) \le 0\ (\lambda \ge 1)$ or $0 < \pi'(0) < \pi(0)$, the effect on after tax profits is always negative (condition (23) holds for any γ). In this case, the analysis of the effects of public expenditure on u^* and g^* is similar to that put forward when public expenditure does not affect productivity. This is because the change in the after tax profit, induced by productivity improvements and wage bargaining, reinforces (or is never strong enough to countervail) the effect of taxation.

Assuming $\pi(0) < \pi'(0) < \infty$, the productivity effect is positive and the effect of public expenditure on after tax profits is nonmonotonic. If $0 \le \gamma < \bar{\gamma}$, after tax profits are affected positively by public expenditure: the productivity effect exceeds the pure public expenditure effect; conversely, if $\gamma > \bar{\gamma}$, after tax profits are affected negatively by public expenditure: the productivity effect is weaker than the pure public expenditure effect. After tax profits are at their peak when $\gamma = \bar{\gamma}$, where $\bar{\gamma}$ solves $\pi'(1-\bar{\gamma}) = \pi(\bar{\gamma})$.

Considering the equilibrium corresponding to the warranted rate of growth compared to the case when productivity is unaffected: public expenditure has still no impact on the degree of capacity utilisation; the warranted rate of growth, instead, changes according to a 'bell shape' (see Figures 9 and 10):

$$\frac{d\tilde{u}}{d\gamma} = 0 \quad \text{and} \quad \frac{d\tilde{g}}{d\gamma} = -s_\pi a\tilde{u}[\pi - \pi'(1-\gamma_L)] \geq (<)\, 0 \quad \text{for} \quad \gamma \leq (>) \overline{\gamma}$$

If $0 \leq \gamma < \overline{\gamma}$, $\tilde{g}$ increases; whereas $\tilde{g}$ decreases for $\gamma > \overline{\gamma}$. It reaches its maximum value at $\gamma = \overline{\gamma}$. The relationship between the size of the government and the equilibrium rate of growth corresponds to the one which holds for Barro's (1990) neoclassical endogenous growth model. For γ sufficiently small the productivity effect (which reduces the costs of production and influences distribution in favours of profits) prevails over the effect of the increase in the government's size that, through taxation, lowers saving at normal capacity utilization. On the contrary, for γ sufficiently large, the pure public expenditure effect dominates the productivity effect.

The effects of public expenditure on the external equilibria, when they exist, are more difficult to disentangle. For the low equilibrium, we have that:

$$\frac{du_L^*}{d\gamma} \geq (<)\, 0 \quad \text{and} \quad \frac{dg_L^*}{d\gamma} = -s_\pi a\tilde{u}[\pi - \pi'(1-\gamma)] + \phi'(u_L^*)\frac{du_L^*}{d\gamma} \geq (<)\, 0 \quad \text{for } \gamma \leq (>) \overline{\gamma}$$

The positive (negative) impact of public expenditure on the degree of capacity utilisation in the low equilibrium can be explained as follows: for a given degree of capacity utilisation, an increase (reduction) in the after tax profit, following the increase in public expenditure, implies a raise (reduction) in saving smaller than a raise (reduction) in investment: consequently, saving falls short of (exceeds) investment. Since for e_L condition (15) holds, in order to equilibrate saving and investment, the degree of capacity utilisation should rise (fall). Moreover, in the low equilibrium, as public expenditure increases and after tax profit rises (falls), the rate of growth increases (decreases) faster than the warranted rate of growth due to the positive (negative) impact on the degree of capacity utilisation.

For the high equilibrium, the impact of public expenditure on the degree of capacity utilization is negative or positive depending on if after tax profits decrease or increase; whereas the rate of growth increases or decreases, depending also on the level of public expenditure, we have that:

if $\gamma < \overline{\gamma}$ then $\dfrac{du_H^*}{d\gamma} < 0$ and $\dfrac{dg_H^*}{d\gamma} = s_\pi a\tilde{u}[\pi(1-\gamma)]' + \phi'(u_H^*)\dfrac{du_H^*}{d\gamma} \geq (<)\, 0$ for $\gamma \leq (>) \widehat{\gamma}_1$ and

if $\gamma > \overline{\gamma}$ then $\dfrac{du_H^*}{d\gamma} > 0$ and $\dfrac{dg_H^*}{d\gamma} = s_\pi a\tilde{u}[\pi(1-\gamma)]' + \phi'(u_H^*)\dfrac{du_H^*}{d\gamma} \geq (<)\, 0$ for $\gamma \leq (>) \widehat{\gamma}_2$, where $\widehat{\gamma}_i$ solves $s_\pi \pi a \tilde{u}(1-\widehat{\gamma}_i) = \phi'(u_H^*)u_H^*$ and where $i = 1, 2$.

The negative (positive) impact of public expenditure on the degree of capacity utilisation in the high equilibrium can be explained as follows: for a given degree of capacity utilisation, the increase (reduction) in the after tax profit, following the increase in public expenditure and the rise in the profit share (as it captures the productivity improvements), implies an increase (reduction) in saving greater that the increase (reduction) in investment: consequently, saving exceeds (falls short of) investment. Since for e_H condition (16) holds, in order to equilibrate saving and investment, the degree of capacity utilisation should fall (rise).

Moreover, in the high equilibrium, when $\gamma < \overline{\gamma}$, the effect of public expenditure on the rate of growth is positive for $\gamma < \widehat{\gamma}_1$ – when the increase in the warranted rate of growth, induced by a rise in the after tax profit, is larger than the reduction of the degree of capacity utilization – and negative for $\gamma > \widehat{\gamma}_1$ – when the increase in the warranted rate of growth is not strong enough to countervail the reduction of the degree of capacity utilization; when $\gamma > \overline{\gamma}$, the effect of public expenditure on the rate of growth is positive for $\gamma < \widehat{\gamma}_2$ – when the decrease of the warranted rate of growth, induced by a fall in the after tax profit, is not strong enough to countervail the rise of the degree of capacity utilisation – and it is negative for $\gamma > \widehat{\gamma}_2$ – when the decrease in the warranted rate of growth dominates the rise in the degree of capacity utilisation. The high equilibrium rate of growth has two relative maxima in correspondence of $\widehat{\gamma}_1$ and $\widehat{\gamma}_2$ (when they exist) and an extremum at $\overline{\gamma}$, where $\widehat{\gamma}_1 \le \overline{\gamma} \le \widehat{\gamma}_2$.

As before, the condition for a positive impact of public expenditure on the equilibrium rate of capital accumulation can be expressed in the terms of partial elasticities:

$$\eta_{\pi(1-\gamma)} < (>)\, \eta_{u^*} \text{ for } \gamma < (>)\, \overline{\gamma},$$

that is, if $\gamma < (>)\, \overline{\gamma}$ the partial elasticity of the rate of capital accumulation with respect to the degree of capacity utilisation should be smaller (larger) than the partial elasticity with respect to the after tax profit. For the low equilibrium and the equilibrium corresponding to the warranted rate of growth, this condition holds for $\gamma < \overline{\gamma}$ (and it is not satisfied for $\gamma > \overline{\gamma}$); whereas for the high equilibrium it holds for $\gamma < \widehat{\gamma}_1 < \overline{\gamma}$ and $\overline{\gamma} < \gamma < \widehat{\gamma}_2$ (and it is not satisfied for $\widehat{\gamma}_1 < \gamma < \overline{\gamma}$ and $\overline{\gamma} < \widehat{\gamma}_2 < \gamma$).

Figure 9 presents surfaces or equilibrium loci which describe the behaviour of the equilibria for the case $b'(\gamma) > 0$ varying public expenditure within $0 \le \gamma \le 0.5$ and the wage-productivity elasticity within $0.6 \le \lambda \le 0.85$ – more specifically, we increase its complement to 1, $(1-\lambda)$, from 0.15 to 0.4.

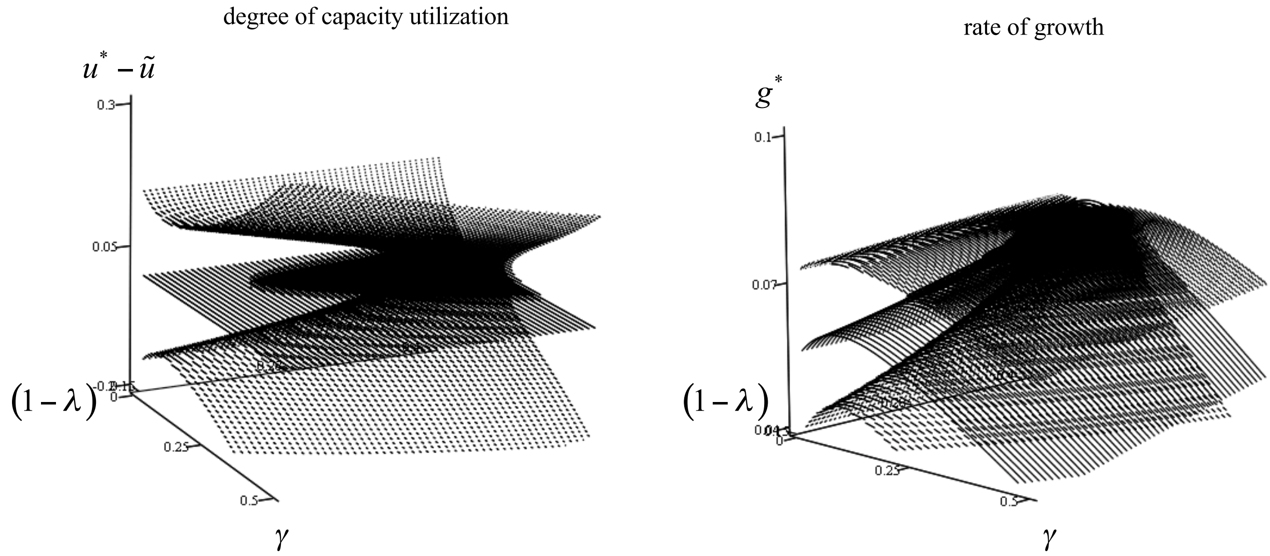


Figure 9

In Figure 10, we study in detail three cases with decreasing wage-productivity elasticity which corresponds to specific sections of the surface plots in Figure 9: that is, (a) $\lambda = 0.8\,(1-\lambda = 0.2)$, (b) $\lambda = 0.635\,(1-\lambda = 0.365)$ and (c) $\lambda = 0.6\,(1-\lambda = 0.4)$.[12] In Figure 10(a) three equilibria exist, the one corresponding to the warranted rate of growth $\tilde{e}$, which is always unstable; the low equilibrium e_L and the high equilibrium e_H which are locally stable for a value of θ sufficiently small. As far as the degree of capacity utilization is concerned, u_L^* increases for $\gamma < \bar{\gamma}$ and decreases for $\gamma > \bar{\gamma}$, with a minimum at $\gamma = \bar{\gamma}$; whereas u_H^* decreases for $\gamma < \bar{\gamma}$ and increases for $\gamma > \bar{\gamma}$, with a maximum at $\gamma = \bar{\gamma}$. The rate of growth, instead, behaves identically in correspondence of all three equilibria, that is, $\tilde{g}$, g_L^* and g_H^* increase for $\gamma < \bar{\gamma}$ and decrease for $\gamma > \bar{\gamma}$; they have a peak at $\gamma = \bar{\gamma}$. We note that for the case represented in Figure 10(a), public expenditure is ineffective in redirect the economy, which is following a low growth path, towards the high equilibrium.

[12] For the other parameters, we use the same combination we used to plot Figure 7 and, for the explicit form of the relationship $\pi(\gamma)$, we take the usual values, that is, $b_0 = 1$, $b_1 = 1.15$, $b_2 = 7.5$ and $w_0 = 0.72$.

Reducing λ strengthens the effect on distribution of a productivity improvement generating more complex phenomena. In Figure 10(b) at $\gamma^P = \bar{\gamma}$ the system undergoes a pitchfork bifurcation: the equilibria e_L and e_H collapse into the equilibrium e_H to re-emerge immediately as γ is increased. Except for the specific value γ^P of public expenditure, the low and the high equilibrium are always locally stable and the warranted growth path is unstable. The behaviour of the degree of capacity utilisation and of the rate of growth along the three equilibria is almost identical as in Figure 10(a). An important exception is the behaviour of the growth rate in the high equilibrium. It increases for $\gamma < \hat{\gamma}_1$ and $\bar{\gamma} < \gamma < \hat{\gamma}_2$ and decreases for $\hat{\gamma}_1 < \gamma < \bar{\gamma}$ and $\gamma > \hat{\gamma}_2$. It has two relative maxima at $\hat{\gamma}_1$ and $\hat{\gamma}_2$ and a minimum at $\bar{\gamma}$.

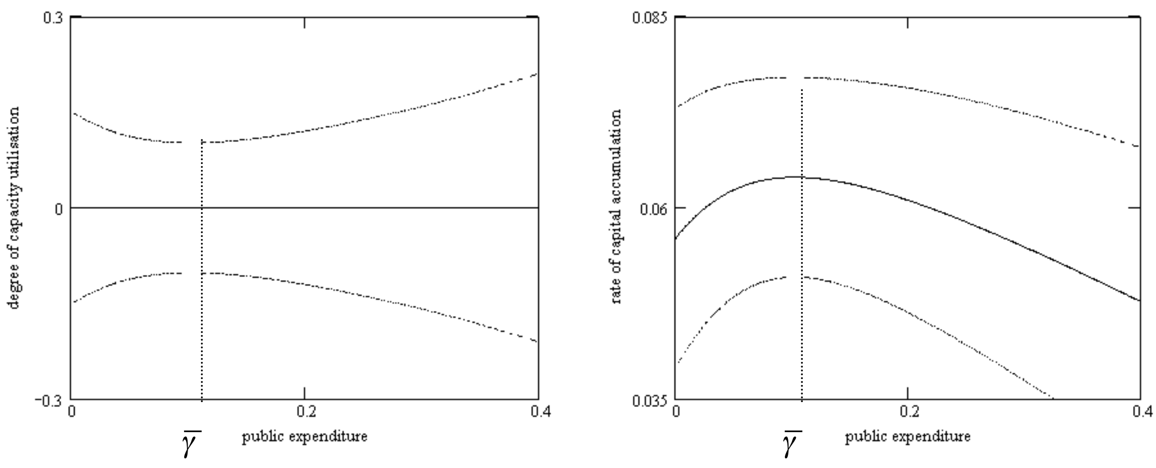
Finally, in Figure 10(c), two distinct pitchfork bifurcations occur at γ^{P1} and γ^{P2}, where $\gamma^{P1} < \bar{\gamma} < \gamma^{P2}$. For $0 \le \gamma < \gamma^{P1}$, three equilibria exists: the low and the high equilibrium are locally stable, whereas the warranted growth path is unstable. In the low equilibrium, the degree of capacity utilisation and the rate of growth increases with public expenditure. The warranted rate of growth also increases within this range. The high equilibrium increases for $0 \le \gamma < \hat{\gamma}_1$ and decreases for $\hat{\gamma}_1 < \gamma < \gamma^{P1}$, with a peak at $\gamma = \hat{\gamma}_1$. As γ crosses the bifurcation value γ^{P1}, the low and the high equilibrium disappear; for $\gamma^{P1} < \gamma < \gamma^{P2}$ only the warranted rate of growth path exists, which is globally stable. It is at its maximum at $\bar{\gamma}$, it increases before and decreases after this value. Increasing further γ, as the bifurcation value γ^{P2} is crossed, the low and high equilibrium re-emerge; at the end of the interval considered, for $\gamma > \gamma^{P2}$, the low and the high equilibrium are locally stable and $\tilde{g}$ is unstable. In the low equilibrium, the degree of capacity utilisation and the rate of growth decreases with public expenditure. The warranted rate of growth also decreases within this range. The high equilibrium increases for $\bar{\gamma} < \gamma < \hat{\gamma}_2$ and decreases for $\gamma > \hat{\gamma}_2$, with a peak at $\gamma = \hat{\gamma}_2$.

Both the cases in Figures 10(b) and 10(c) suggest that there is a scope for public expenditure to shift the economy from a low to a high equilibrium even in the Classical-Harrodian case. In particular, if the economy is located at γ^P in Figure 10(b) or at γ^{P1} or γ^{P2} in Figure 10(c), a suitable small shock could bring it on the high equilibrium.

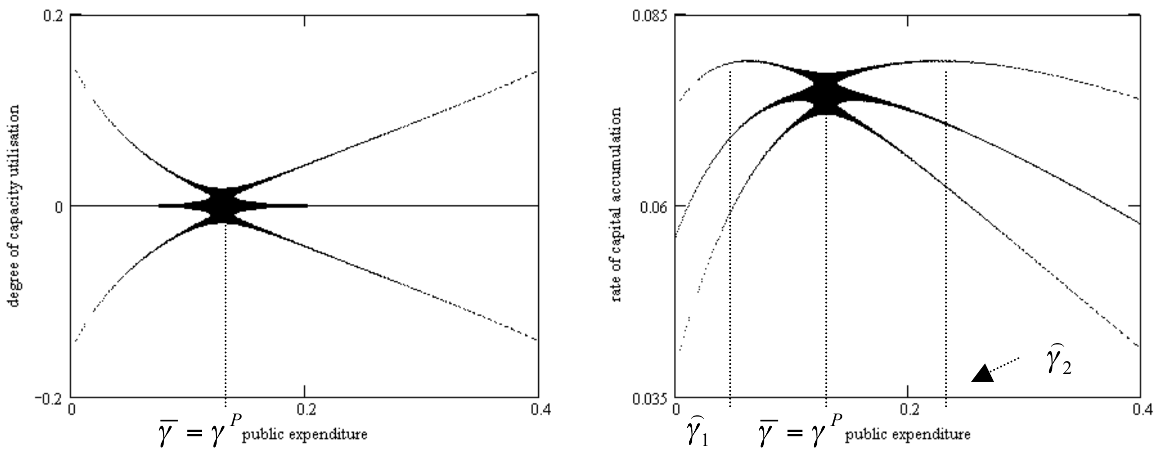
In conclusion, Figures 9 and 10 suggest that, as γ rises there are three different regimes of growth which the economy could experience depending on the initial condition and on the level of public expenditure. Within each regime, the direction of the degree of capacity utilisation and of the rate of growth is reversed when the after tax profit also change its direction (from increasing to decreasing) as public expenditure crosses the value $\bar{\gamma}$. In the first regime (profit led), $\gamma < \bar{\gamma}$, the degree of capacity utilisation is constant or increasing and the rate of growth in also increasing; and for $\gamma > \bar{\gamma}$ the degree of capacity utilisation is constant or falling and the rate of growth is also falling. Such a regime occurs when the economy moves along the lower equilibrium or along the warranted growth path; in the second regime (also profit-led), for $\gamma < \bar{\gamma}$, the degree of capacity utilisation decreases, whereas the rate of growth increases; and for $\gamma > \bar{\gamma}$, the degree of capacity utilisation rises, whereas the rate of growth falls. Such a regime occurs when the economy moves along the high equilibrium and $0 \le \gamma < \hat{\gamma}_1$ and $\gamma > \hat{\gamma}_2$. Finally, in the third regime (public expenditure led), for $\gamma < \bar{\gamma}$, the degree of capacity utilisation and the rate of growth are both falling; and for $\gamma > \bar{\gamma}$, the degree of capacity utilisation and the rate of growth are both rising. Such a regime occurs when the economy moves along the high equilibrium and $\hat{\gamma}_1 < \gamma < \hat{\gamma}_2$. The third regime is the only one for which the paradox of costs hold: the rate of growth and the degree of capacity utilisation both moves in the opposite direction of the after-tax profit.[13]

[13] A fourth growth regime, which is both public expenditure led and wage led, could occur when $\pi'(\gamma) \le 0 \; (\lambda \ge 1)$.

(a)



(b)



(c)

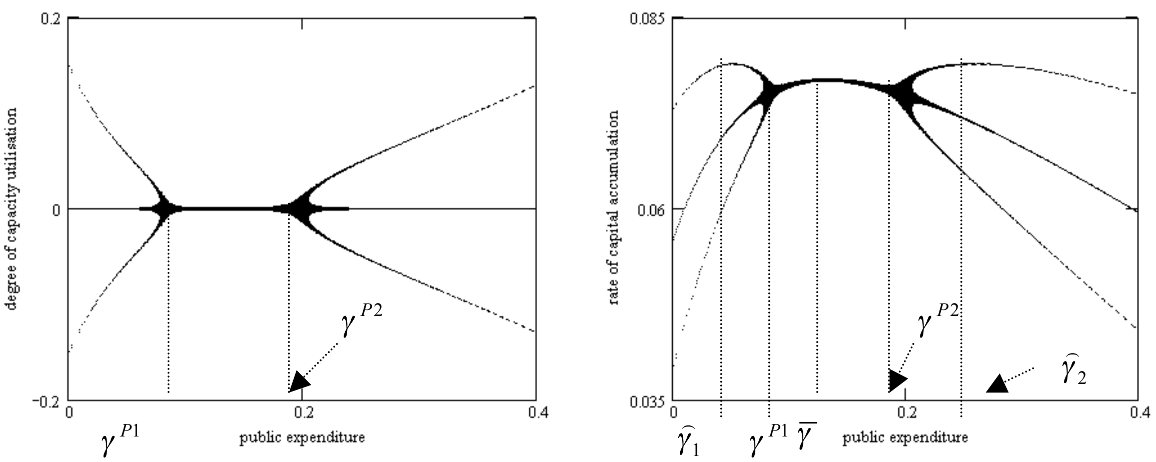


Figure 10

4.3 Dynamic analysis

In this section, we let vary the parameter θ, which represents the speed at which capacity utilization adjusts to excess demand. We first study how the long term behaviour of the economy changes when this parameter is increased. We also explore how public expenditure impacts on such behaviour.

The central equation of the dynamic analysis is the difference equation or map (13) that, for the Classical-Harrodian case, we explicit as follows:

$$\psi(u) = u + \theta\left[\phi(u) - s_\pi \pi a(1-\gamma)(u-\tilde{u})\right]$$

A crucial property of this map is that it is symmetric around the equilibrium $\psi(\tilde{u}) = \tilde{u}$. Such a property reduces the complexity of the dynamic behaviour compared to the Kaleckian case.

In the Appendix, we show that a period doubling sequence to complex behaviour occurs simultaneously for the low and the high equilibrium, u_L^* and u_H^*. We also show that the global stability properties, which increase in complexity as θ is increased, follows a symmetric pattern. These phenomena are typical to the class of symmetric bimodal maps, to which $\psi(u)$ belongs for values of θ sufficiently high.

Moreover, we describe in analytical detail what follows: in the first place, for a value of γ which gives rise to three equilibria, as θ is increased, two coexisting attractors of various periodicity and chaotic attractors emerge: the first attractor cycling around the low equilibrium and the second cycling around the high equilibrium. The two attractors are symmetric to each other and enjoy the same periodicity. In the second place, as for the Kaleckian case, the structure of the basins of attraction of the two attractors becomes more complicated as θ varies, even though it follows a symmetric pattern. Finally, for θ sufficiently large, a global bifurcation takes place: however its impact on the long period behaviour of the economy is less catastrophic than for the one which occurs for the Kaleckian case.

We now explore, albeit briefly, the impact of a change in γ on the dynamic properties of the map $\psi(u)$. Concerning the effect on the local stability properties of the fixed points. For the case in which public expenditure does not affect the production coefficients, we have that

$$\frac{d\theta^F}{d\gamma} = \theta^F \frac{s_\pi \pi a + \phi''(u^*)\, du^*/d\gamma}{s_\pi \pi a(1-\gamma) - \phi'(u^*)}.$$

Since $\phi''(u_L^*) > 0$ and $\frac{du_L^*}{d\gamma} < 0$, for the low equilibrium and $\phi''(u_H^*) < 0$ and $\frac{du_H^*}{d\gamma} > 0$ for the high equilibrium, it follows that the sign of $\frac{d\theta^F}{d\gamma}$ is indeterminate. That is, the effect of public expenditure on the local stability is not univocal.

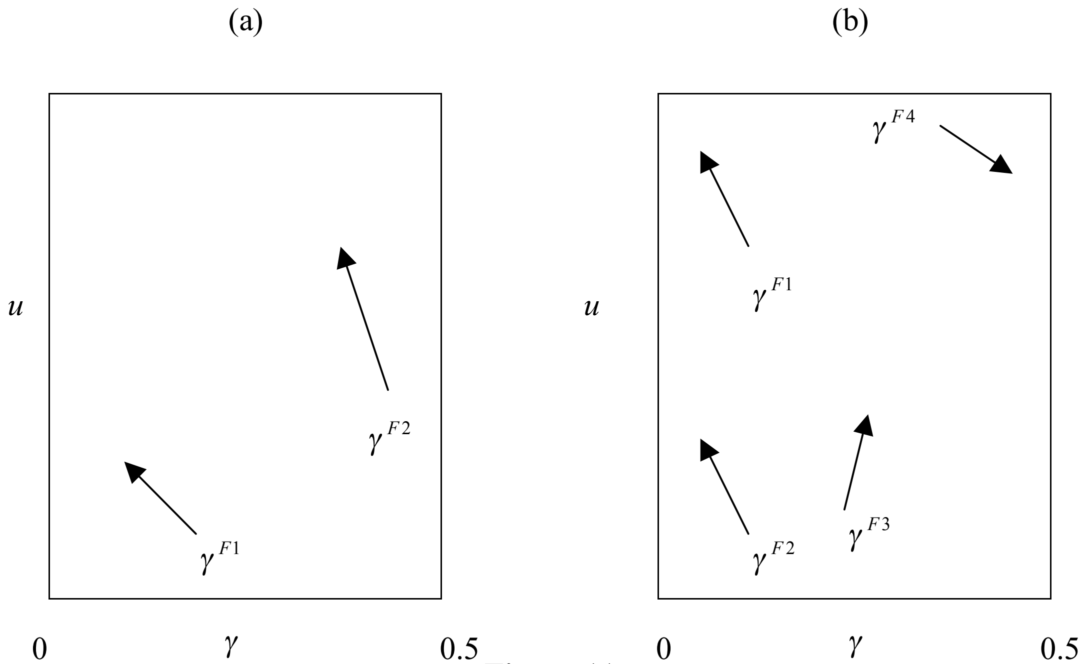


Figure 11

Figure 11(a), which deals with the case in which public expenditure does not affect the production coefficients, presents a bifurcation diagram showing the impact of $0 \le \gamma \le 0.5$ on the long term behaviour of the degree of capacity utilisation for $\theta = 40$.[14] In Figure 11(a), two Flip bifurcations are visible for the high equilibrium and, by symmetry, for the low equilibrium: γ^{F1} and γ^{F2}. The external equilibria are stable for $0 \le \gamma < \gamma^{F1}$. They lose stability at γ^{F1} and two locally attracting period two cycle emerge (of which only the one cycling around the high equilibrium is visible). Increasing further public expenditure as γ crosses γ^{F2}, the two period two cycle disappear and the external equilibria become again locally attracting in the interval $\gamma^{F2} < \gamma \le 0.5$.

[14] To plot Figure 11(a) we used, for the other parameters, $\tilde{u} = 0.5$, $a = 0.5$, $s_\pi = 0.8$, $\beta_1 = 0.02$, $\beta_2 = 7.5$, $b_0 = 1$, $b_1 = 0$, $b_2 = 7.5$, $\lambda = 0.7$ and $w_0 = 0.72$. Moreover, we set as initial condition $u_0 = 0.51$.

Figure 11(a) shows that public expenditure has, for low values of public expenditure, a destabilising effect and, for high values of public expenditure a stabilising effect. This applies both on the local and on the global stability of the economic system.

For the case in which public expenditure affects labour productivity, we have that

$$\frac{d\theta^F}{d\gamma} \equiv \theta^F \frac{s_\pi a\left[\pi(\gamma) - \pi'(\gamma)(1-\gamma)\right] + \phi''(u^*)\frac{du^*}{d\gamma}}{s_\pi \pi(\gamma) a(1-\gamma) - \phi'(u^*)}$$

When condition (23) holds, i.e. $\pi - \pi'(\gamma)(1-\gamma) > 0$, since $\phi''(u_L^*) > 0$ and $\frac{du_L^*}{d\gamma} < 0$, for the low equilibrium and $\phi''(u_H^*) < 0$ and $\frac{du_H^*}{d\gamma} > 0$, for the high equilibrium, it follows that the sign of $\frac{d\theta^F}{d\gamma}$ is indeterminate. Moreover, when condition (24) holds, i.e. $\pi - \pi'(\gamma)(1-\gamma) < 0$, since $\phi''(u_L^*) > 0$ and $\frac{du_L^*}{d\gamma} < 0$, for the low equilibrium, and $\phi''(u_H^*) < 0$ and $\frac{du_H^*}{d\gamma} < 0$, for the high equilibrium, it follows again that the sign of $\frac{d\theta^F}{d\gamma}$ is not univocal.

Figure 11(b), which deals with the case in which public expenditure affects labour productivity, presents a bifurcation diagram showing the impact of $0 \le \gamma \le 0.5$ on the long term behaviour of the degree of capacity utilisation for $\theta = 40$.[15] In Figure 11(b) , four Flip bifurcations are visible for the high equilibrium and, by symmetry, for the low equilibrium: two at the left of $\bar{\gamma}$, i.e., γ^{F1} and γ^{F2}, and two at the right of $\bar{\gamma}$, i.e. γ^{F3} and γ^{F4}. The external equilibria are locally stable for γ belonging to the intervals $0 \le \gamma < \gamma^{F1}$, $\gamma^{F2} < \gamma < \gamma^{F3}$ and $\gamma^{F3} < \gamma \le 0.5$. They lose local stability at γ^{F1} and at γ^{F3} and gain stability at γ^{F2} and at γ^{F4}. Two attracting period two cycles exist (of which only the one cycling around the high equilibrium is visible), for γ belonging to the intervals $\gamma^{F1} < \gamma < \gamma^{F2}$ and $\gamma^{F3} < \gamma < \gamma^{F4}$.

Figure 11(b) shows that the external equilibria are stable for very low and for very high values of public expenditure and for values sufficiently close to $\bar{\gamma}$. As far as global stability is concerned,

[15] To plot Figure 11(b) we used, for the other parameters, $\tilde{u} = 0.5$, $a = 0.5$, $s_\pi = 0.8$, $\beta_1 = 0.02$, $\beta_2 = 7.5$, $b_0 = 0.5$, $b_1 = 1.12$, $b_2 = 7.5$, $\lambda = 0.7$ and $w_0 = 0.72$. Moreover, we set as initial condition $u_0 = 0.51$.

public expenditure is destabilising starting close to zero and it is stabilising at a high levels of γ. It is also stabilising when it moves towards $\bar{\gamma}$ (both from the left and from the right).

5. Conclusions

The analysis presented in this paper points out how a change in Government expenditure and its composition can affect the rate of growth of the economy. These effects work through a transfer of income from the private to the Government sector and through a re-distribution of income between the working and the capitalist class.

As far as the comparison of steady growth equilibria is concerned, the Kaleckian interpretation of the model here presented allows the achievement of more clear-cut results than the Classical-Harrodian interpretation.

In the Kaleckian interpretation a change in the "unproductive" Government expenditure only generates a transfer of income between the private and the Government sector. It occurs a "paradox of costs" applied to the Government sector, because the propensity to spend of this sector is higher than the propensity to consume of the capitalist class. Thus, a reduction in the net profit of the capitalist class, due to the higher tax rate, is accompanied by a higher level of effective demand and a higher rate of growth of the economy. Obviously this result only applies to the stable equilibria of the model.

Another comparative statics result of this case shows that an increase in Government expenditure can lead the economy out of a "poverty trap", towards an equilibrium with a higher rate of growth, with an hysteresis effect that prevents the economy from going back to the poverty trap.

A change in the "productive" government expenditure generates both a transfer of income from the private to the government sector and a re-distribution of income between the two classes, which depends on how the increase in productivity is divided between them. The traditional "paradox of costs", related to the re-distribution between the classes, now occurs in addition to that applied to the government sector. If the increase in productivity appropriated by the capitalist class is smaller than the reduction in profits generated by the higher tax rate, then the effective demand and the rate of growth rise. The simultaneous and opposite effects of the increase in productivity and in the taxation allows the identification of a level of government expenditure that maximises the after tax profit and minimises the rate of growth.

Moreover, for low levels of government expenditure, if the increase in productivity causes a significant increase in the after tax profits, the economy can experience a dramatic fall in the degree of capital utilisation and the rate of growth. Further increases in government expenditure can reduce the after tax profits and the rate of growth. If the increase in productivity does not cause a significant increase in the after tax profits the economy can experience a hysteresis effect from the low to the high equilibrium.

In the Classical-Harrodian intepretation the effects of an increase in the government expenditure tend to be dominated by those on the warranted rate of growth, which coincides with the central equilibrium.

The "unproductive" expenditure affects negatively the warranted rate and the central equilibrium of the model. As to the other equilibria we can have that when $u^* < \tilde{u}$, the rate of growth reduces even more because of the further reduction in the utilization rate. Unlike what happens in the Kaleckian interpretation, the increase in the government expenditure worsens the low equilibrium of the economy pushing it in an poorer position. If, instead, $u^* > \tilde{u}$, the rate of growth increases when the investment function is more elastic with respect to the utilization rate than with respect to the after tax profit. The 'paradox of costs' holds in this case too. Moreover, the basins of attraction of the two external equilibria are symmetrical and their relative size is not affected by the government expenditure.

The "productive" expenditure instead can affect the warranted rate changes according to a "bell shape" curve. The simultaneous and opposite effects of the increase in productivity and in the taxation allows the identification of a level of government expenditure that maximises the after tax profit and, unlike in the Kaleckian interpretation, the rate of growth too. Like in Barro (1990) there is an optimal dimension of the government sector.

As to the other equilibria, the low equilibrium follows the trend of the warranted rate, rising at a higher rate when the latter rises and diminishing at a higher rate when the latter falls. The high equilibrium follows a more complex trend. When the dimension of the government sector is below optimal, an increase in the expenditure raises the after tax profits and reduces the rate of growth if the investment function is more elastic with respect to the utilisation rate than with respect the after tax profit: the "paradox of costs" occurs. When instead the size of the government sector is above optimal, an increase in the expenditure reduces the after tax profit and raises the rate of growth if the investment function is more elastic with respect to the utilisation rate than with respect to the after tax profit. Thus the “paradox of costs” occurs again.

As to the dynamics, both interpretations exhibit a high degree of complexity.

In the Kaleckian interpretation, if the variation of the degree of capital utilisation in the face of an excess of demand is “relatively small”, the convergence of the economy to the “low” or “high” equilibrium depends on the initial conditions. If the economy moves from above (below) the middle equilibrium, it converges towards the high (low) equilibrium. Moreover, an increase in the government expenditure widens the basins of attraction of the high equilibrium, while that of the low equilibrium shrinks, because the intermediate equilibrium is associated with a lower rate of growth, increasing the probability that the economy converges to a high equilibrium. If the variation of the degree of capital utilisation in the face of an excess of demand is “relatively large”, a period doubling sequence to complex behaviour could occur both for the low equilibrium and the high equilibrium. Two coexisting attractors of various periodicity or chaotic emerge: the first cycling around the low and the second around the high equilibrium. The two attractors are asymmetric to each other and with different periodicity. The structure of the basins of attraction also increases in complexity becoming more and more disconnected and intermingled. Hysteresis phenomena of a dynamic nature may emerge and even structural changes could take place. Thus, the effect of public expenditure on the global dynamics becomes difficult to predict.

The local stability analysis results are neater. When public expenditure is ‘unproductive’ it has a stabilising effect on the low equilibrium. It has also a stabilising effect on the high equilibrium as long as its level is sufficiently high. For the case of ‘productive’ public expenditure it could be both stabilising or destabilising for the low equilibrium, depending on parameter values; whereas it is destabilising for the high equilibrium when its level is sufficiently low or sufficiently high.

In the Classical-Harrodian interpretation, if the variation of the degree of capital utilisation in the face of an excess of demand is “relatively small”, the convergence of the economy to the “low” or “high” equilibrium also depends on the initial conditions. If the economy moves from above (below) the equilibrium corresponding to the warranted rate of growth, it converges towards the high (low) equilibrium. However, government expenditure has no effect on the size of the basins of attraction. The probability that the economy converges to a high equilibrium or to a low equilibrium does not change. As for the Kaleckian interpretation, if the variation of the degree of capital utilisation in the face of an excess of demand is “relatively large”, the long term behaviour of the system also increases in complexity: a period doubling sequence to complex behaviour could occur both for the low and the high equilibrium. Two coexisting attractors of various periodicity or chaotic emerge: the first cycling around the low and the second around the high equilibrium. However, differently from the previous case, the two attractors are symmetric to each other and enjoy the same periodicity. The basins of attraction becomes more and more disconnected and intermingled between them, but they still keep a symmetric structure. Hysteresis phenomena of a dynamic nature may emerge and even structural changes could take place, even though with a less catastrophic impact on the economy compared to the Kaleckian interpretation.

For the Classical-Harrodian interpretation, simulations show that public expenditure has a locally stabilising effect on the high equilibrium and, by symmetry, on the low equilibrium as long as its level is sufficiently low or sufficiently high. This result applies for both cases of 'unproductive' and 'productive' public expenditure.

From our analysis it follows that the adoption of a discrete time framework and the introduction of a nonlinearity in the investment function allow the emerging of a wide range of complex phenomena which cannot occur in standard post Keynesian linear models of steady growth analysis. The occurrence of such phenomena causes the long-run behaviour of the economy to be difficult to predict, justifying on a sound theoretical basis the long-run discrepancy between the normal and the current degree of capacity utilisation.

APPENDIX

In this appendix, we describe in analytical detail the global dynamics properties of the map $\psi(u)$ for the Kaleckian and the Classica-Harrodian cases.

We need to introduce a few definitions. A trajectory or orbit is a ordered sequence $(u_0,u_1,u_2,...,u_n,...)$, where $u_1=\psi(u_0)$, $u_n=\psi^n(u_0)$ and where $\psi^n(u_0)=\psi(\psi^{n-1}(u_0))$ is the n[th] iterate of u_0. A backward orbit corresponds to $(u_0,u_{-1},u_{-2},...,u_{-n},...)$, where $u_{-1}=\psi^{-1}(u_0)$, $u_{-n}=\psi^{-n}(u_0)$ and where $\psi^{-n}(u_0)=\psi^{-1}(\psi^{-(n-1)}(u_0))$ is the n[th] pre-iterate of u_0. Each initial condition has only one orbit and as many backward orbits as its pre-images. A *k*-periodic orbit or period-*k* cycle is a perpetually repeated sequence; for example, a 3-period cycle is concisely defined by (u_0^*,u_1^*,u_2^*). *k* the smallest number of iterations necessary for the repetition to take place. A *k*-periodic point u^* is one of the elements of a period *k* cycle, where $u^*=\psi_k(u^*)$. A fixed point (a long-run equilibrium) is a periodic point of order 1, $u^*=\psi(u^*)$. An attractor is the limiting set of a trajectory. A basin of attraction is the set of initial values whose trajectories sooner or later converge to such attractor. A periodic attractor is an attracting periodic orbit. A chaotic attractor is an attracting aperiodic orbit.

A.1 The Kaleckian case

For our study concerning the Kaleckian case, we assume the following parameters combination: $\alpha = 0.07$, $\tilde{u}=0.5$, $a = 0.5$, $s_\pi=0.8$, $\beta_1=0.02$, $\beta_2=15$, $w_0=0.72$, $b_0=2$, $b_1=0$, $b_2=7.5$ and $\lambda = 0.7$.

Figure A.1 plots the map $\psi(u)$ for $\theta = 5$ (dashed line), $\theta = 15$ (dotted line) and $\theta = 30$ (solid line) and for $\gamma=0.24$ and $\pi\cong 0.415$.

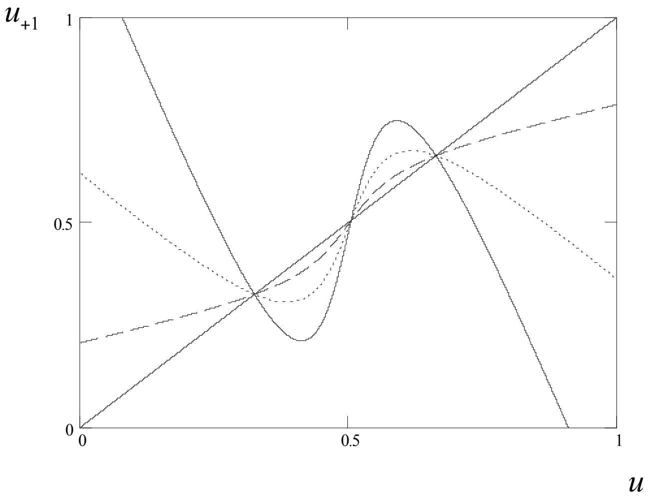


Figure A.1

For $\theta = 0$, the map $\psi(u)$ coincides with the 45° line, $\psi'(u) = 1$. As shown in Figure A.1, increasing θ the steepness of the map increases until two critical points (extrema), c^m and c^M, are formed. That is, for θ sufficiently high, $\psi(u)$ belongs to the class of bimodal maps.

θ does not affect the fixed points of the map (i.e. the long-run steady growth equilibria). To evaluate the dynamic properties of the map $\psi(u)$, we need to consider three cases.

In the first case, obtained by setting γ sufficiently small (in our study, $\gamma = 0$), there exists only the equilibrium u_L^*. Defining $\underline{u} = \min(\overline{u},1)$, we disregard the orbits that start outside the interval $(0, \underline{u})$ since they may exit the (0, 1) interval, where $\overline{u}$ is the value of u at which $\psi(u)$ cuts the horizontal axis. For θ sufficiently small but larger than 0, $\overline{u} > \psi(0)$.

As shown in Figure A.1, θ affects (positively) $\psi(0)$, with $\psi(0) = 0$ for $\theta = 0$. Moreover, θ also impacts (negatively) on $\overline{u}$, when the latter exists.

For $0 < \theta < \theta_L^F$, all the orbits converge to u_L^*, which is globally attractive: the orbits converge monotonically for $\psi'(u_L^*) > 0$ or with oscillations when $\psi'(u_L^*) < 0$.

Increasing θ, when θ crosses θ_L^F, u_L^* loses stability via a Flip or period doubling bifurcation, where $\theta_L^F \equiv \dfrac{2}{\left[s_\pi \pi a(1-\gamma) - \phi'(u_L^*)\right]}$. An attracting period two cycle emerges (u_0^L, u_1^L). Increasing further θ, the period two cycle becomes repelling and a stable period four emerges.

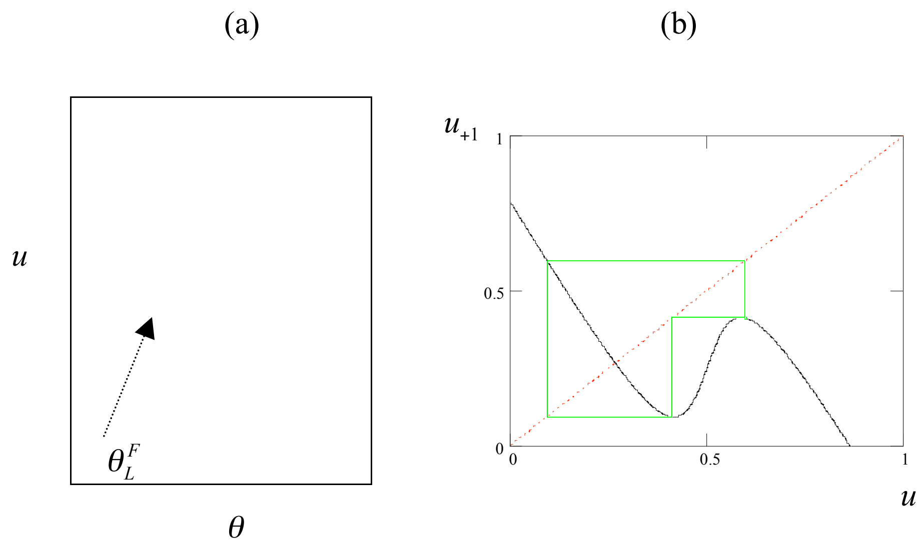


Figure A.2

As shown in Figure A.2(a), plotted for $\gamma = 0$, $u_0 = 0.5$ and $11 \le \theta \le 21$, a period doubling route to chaos takes place. In addition, a period-three window – the hallmark of chaos, as confirmed by the Li-Yorke theorem – is easily visible. The specific period-three corresponding to $\theta = 19$ is shown in Figure A.2(b).

When $\psi(0) = \bar{u}$, a repelling period 2 cycle (υ_0, υ_1) enters the (0, 1) interval, with $\upsilon_0 = 0$ and $\upsilon_1 = \bar{u}$. Increasing θ this repelling period 2-cycle shrinks, i.e. $\upsilon_0 > 0$ and $\upsilon_1 < \bar{u}$. When $\psi(0) > \bar{u}$, if u_0 starts within the interval (υ_0, υ_1), the system converges to an economically meaningful (periodic or aperiodic) attractor $0 < u \le 1$. It diverges to $\pm\infty$ otherwise.

We now consider the case in which three equilibria exist: for our study we set $\gamma = 0.2$ and keep the parameters combination used above.

For θ sufficiently small, there are two locally attracting fixed points u_L^* and u_H^*. For $(u_I^*)_{-1}^l < 0$, the repelling fixed point u_I^* separates the interval $(0,1)$ into two basins, where $(u_I^*)_{-1}^l < 0$ represents the left pre-iterate of u_I^*. The interval $(0, u_I^*)$ is the basin of attraction of u_L^*, whereas $(u_I^*, 1)$ represents the basin of attraction of u_H^*. For $(u_I^*)_{-1}^l > 0$, the basin of attraction of u_H^* expands, including a piece of the basin of attraction of u_L^*. The new basin of attraction of u_L^* corresponds to the union of two disjoint intervals $(0, (u_I^*)_{-1}^l) \cup (u_I^*, 1)$; it follows that the basin of attraction of u_L^* shrinks to $((u_I^*)_{-1}^l, u_I^*)$. For $(u_I^*)_{-1}^r < 1$ also the basin of attraction of u_L^* expands and becomes disconnected, corresponding to $((u_I^*)_{-1}^l, u_I^*) \cup ((u_I^*)_{-1}^r, 1)$, where $(u_I^*)_{-1}^r$, represents the right pre-iterate of u_I^*. The basin of attraction of u_H^* reduces to $(0, (u_I^*)_{-1}^l) \cup (u_I^*, (u_I^*)_{-1}^r)$.

When θ crosses the value θ_H^F, u_H^* loses stability and an attracting period-two cycle emerges (u_0^H, u_1^H). See figure A.3(a), plotted for $16 < \theta < 32$ and $u_0 = 0.51$.

For $\psi(0) > (u_I^*)_{-1}^r$, the basin of attraction of u_L^* changes further becoming

$$(0, (u_I^*)_{-2}^r) \cup ((u_I^*)_{-1}^l, u_I^*) \cup ((u_I^*)_{-1}^r, 1),$$

where $(u_I^*)_{-2}^r$ is the second right pre-iterate of u_I^*. The two pieces set $((u_I^*)_{-2}^r, (u_I^*)_{-1}^l) \cup \left(u_I^*, (u_I^*)_{-1}^r\right)$, is the basin of attraction of the period-two orbit cycling around u_H^*.

Increasing θ further, the period two cycle loses stability and a locally stable period four cycle emerges. See figure A.3(a).

When $(u_I^*)_{-2}^l > \psi(1)$, the basin of attraction of u_L^* shrinks becoming

$$(0, (u_I^*)_{-2}^r) \cup ((u_I^*)_{-1}^l, u_I^*) \cup ((u_I^*)_{-1}^r, (u_I^*)_{-2}^l),$$

where $(u_I^*)_{-2}^l < 1$ is the second right pre-iterate of u_I^*. The disconnected set

$$((u_I^*)_{-2}^r, (u_I^*)_{-1}^l) \cup \left(u_I^*, (u_I^*)_{-1}^r\right) \cup \left((u_I^*)_{-2}^l, 1\right)$$

is the basin of attraction of the period four orbit cycling around u_H^*.

For higher values of θ, cycles of any order and even aperiodic orbits emerge, cycling around u_H^*. It is possible to identify a chaotic attractor 'trapped' within the set $(c_1^M, c^M) \subset \left(u_I^*, (u_I^*)_{-1}^r\right)$. See figure A.3(b) plotted for $\theta = 0.24$ and $u_0 = 0.1$.

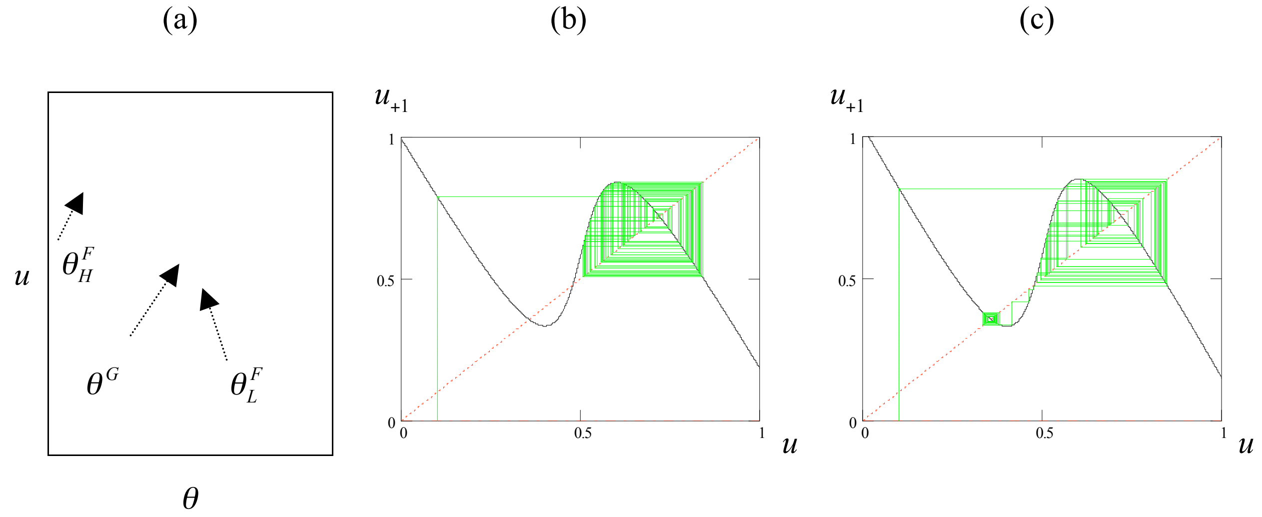


Figure A.3

With higher θs the basin of attractions change further. When $(u_I^*)_{-2}^l < \psi(1)$, the basin of u_L^* shrinks further becoming

$$((u_I^*)_{-3}^l, (u_I^*)_{-2}^r) \cup ((u_I^*)_{-1}^l, u_I^*) \cup ((u_I^*)_{-1}^r, (u_I^*)_{-2}^l),$$

where $(u_I^*)_{-3}^l$ is the third left preiterate of u_I^*. The four pieces disconnected set

$$(0, (u_I^*)_{-3}^l) \cup ((u_I^*)_{-2}^r, (u_I^*)_{-1}^l) \cup \left(u_I^*, (u_I^*)_{-1}^r\right) \cup \left((u_I^*)_{-2}^l, 1\right)$$

is the basin of attraction of the (periodic or aperiodic) attractor cycling around u_H^*.

Then when $\psi(0) > 1$, the latter basin of attraction should be modified into

$$(\hat{u}, (u_I^*)_{-3}^l) \cup ((u_I^*)_{-2}^r, (u_I^*)_{-1}^l) \cup \left(u_I^*, (u_I^*)_{-1}^r\right) \cup \left((u_I^*)_{-2}^l, 1\right),$$

since all trajectories starting within (0, $\hat{u}$) leave the (0, 1) interval after one iteration, where $\hat{u}$ is such that $\psi(\hat{u}) = 1$.

At $c_1^M = u_I^*$, corresponding to $\theta = \theta^G$, the system undergoes a substantial qualitative change, that is, a global bifurcation occurs. The interval (c_1^M, c^M) partially overlaps with the basin of attraction of u_L^*: almost all trajectories that start in (c_1^M, c^M) and, therefore, almost all trajectories in the interval $(\hat{u}, 1)$ converges to u_L^*. See figure A.3(c), plotted for $\theta = 0.25$ and $u_0 = 0.1$.

Increasing further θ u_L^* loses stability and a stable two cycle emerges.

When $\psi(1)=\hat{u}$, a repelling period 2 cycle (υ_0,υ_1) enters the (0, 1) interval, where $\upsilon_0=\hat{u}$ and $\upsilon_1=1$. Increasing further θ this period 2-cycle shrinks, with $\upsilon_0>\hat{u}$ and $\upsilon_1<1$. For $\psi(1)<\hat{u}$, the system converges to an economically meaningful attractor, i.e. $0<u\le 1$, if u_0 starts within the interval (υ_0,υ_1). It diverges to $\pm\infty$ otherwise.

For $\theta>\theta_L^F$, u_L^* loses stability and an attracting period two cycle emerges , increasing θ cycles of any order and aperiodic behaviour emerges, where $\theta_L^F\equiv\dfrac{2}{\left[s_\pi\pi a(1-\gamma)-\phi'(u_L^*)\right]}$ See figure A.4(a).

When $c_1^M<\hat{u}$ almost all trajectories in the interval $(\overline{u},c^m)\cup(c_1^m,1)$ exit the (0, 1) interval, whereas the trajectories that start at (c^m,c_1^m) are trapped in that interval.

Finally when $c_1^m>u_I^*$, also the trajectories starting in (c^m,c_1^m) exit the (0, 1) interval.

The existence of a global bifurcation point could result in unexpected consequences for a policy maker. In particular, a small variation in public expenditure could induce a substantial change in the long-term behaviour of economy. For example, given the above parameter constellation, for $\gamma=0.194$, the economy is settled on a chaotic trajectory cycling around u_H^*, with a long-run average value of u approximately equal to 0.65. Increasing public expenditure to $\gamma=0.195$ induces a global bifurcation letting the economy to plunge into a period-two orbit cycling around u_L^*, with a long-run average value of u approximately equal to 0.355.

Finally, we consider the case, obtained by setting γ sufficiently high (in our study we set $\gamma=0.25$) in which only the equilibrium u_H^* exists. Defining $\underset{\smile}{u}=\max(0,\hat{u})$, we disregard the orbits that start outside the interval $(\underset{\smile}{u},1)$ since they may exit the (0, 1) interval.

For $0<\theta<\theta_H^F$, all the orbits converge to the unique fixed point u_H^*, which is globally attracting: the orbits converge monotonically when $\psi'(u_H^*)>0$ and with oscillations when $\psi'(u_H^*)<0$.

Increasing θ, when θ crosses θ_H^F, u_H^* loses stability via a Flip bifurcation, an attracting period two cycle emerges (u_0^L,u_1^L). Increasing further θ, a period doubling route to chaos takes place.

Finally, when θ is such that $c^M > 1$, almost all trajectories exit the (0, 1) interval.

A.2 The Classical-Harrodian case

In this section of the appendix, we describe in analytical detail the global dynamics properties of the map $\psi(u)$ for the Classical-Harrodian case.

For our study we assume the following parameters combination: $\tilde{u} = 0.5$, $a = 0.5$, $s_\pi = 0.8$, $\beta_1 = 0.02$, $\beta_2 = 7.5$, $w_0 = 0.62$, $b_0 = 1$, $b_1 = 0$, $b_2 = 7.5$ and $\lambda = 0.7$.

Figure A.4 plots the map $\psi(u)$ for $\theta = 10$ (dashed line), $\theta = 30$ (dotted line) and $\theta = 60$ (solid line) and for $\gamma = 0.24$ and $\pi \cong 0.38$.

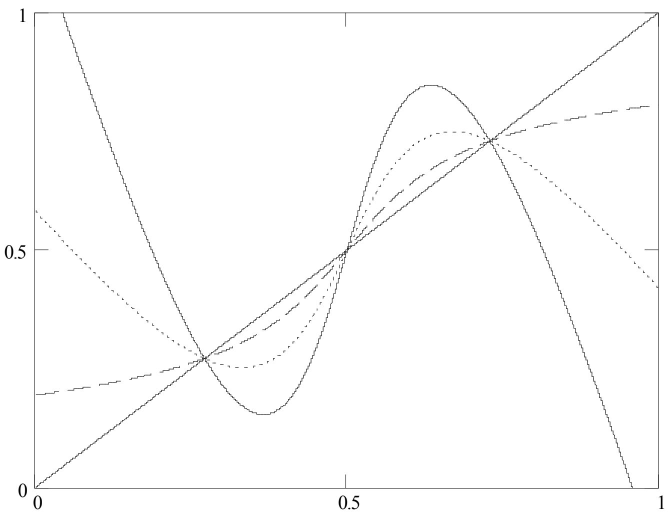


Figure A.4

For $\theta = 0$, the map $\psi(u)$ coincides with the 45° line, $\psi'(u) = 1$. As shown in Figure A.1, increasing θ the steepness of the map increases until two critical points (extrema), c^m and c^M, are formed, where $c^m = 1 - c^M$. That is, for θ sufficiently high, $\psi(u)$ belongs to the class of bimodal maps.

For the Classical-Harrodian case, the map $\psi(u)$ is symmetric around the equilibrium $\tilde{u}$. Defining $\underline{u} = \min(\overline{u}, 1)$ or, equivalently, $(1 - \underline{u}) = \max(0, 1 - \overline{u})$, we disregard the orbits that start outside the interval $(1-\underline{u}, \underline{u})$ since they may exit the (0, 1) interval, where $\overline{u}$ $(1-\overline{u})$ is the value of u at which $\psi(u)$ cuts the horizontal axis (the horizontal line at 1).

When only the equilibrium $\psi(\tilde{u}) = \tilde{u}$ exists, all trajectories starting inside the admissible interval $(1-\underline{u}, \underline{u})$, converges to the unique fixed point. In our simulation study, this case occurs, for example, at $\gamma = 0$.

When three equilibrium exist, u_L^*, $\tilde{u}$ and u_H^*, the external equilibria are stable for

$$\theta < \theta^F \equiv \frac{2}{\left[s_\pi \pi a(1-\gamma) - \phi'(u^*)\right]}$$

where the bifurcation value θ^F is equal for both equilibria since $\phi'(u_L^*) = \phi'(u_H^*)$.

As θ is increased, the structure of the basins of attraction becomes more and more disconnected and intermingled. Their structure changes as follows:

Each time the condition $\psi(0) = \psi_j^{-i}(\tilde{u})$ is violated in the direction of $\psi(0) > \psi_j^{-i}(\tilde{u})$, where i = 1, 2, …, n and $j = l$ for i odd and $j = r$ for i even; or, by symmetry, the condition $\psi(1) = \psi_k^{-i}(\tilde{u})$ is violated in the direction of $\psi(1) < \psi_k^{-i}(\tilde{u})$ where i = 1, 2, …, n and $k = r$ for i odd and $k = l$ for i even, the basins of attraction of u_L^* and u_H^* (or of other attractors with different periodicity or aperiodic) becomes more disconnected and intermingled. In particular, for i odd a piece of the basin of attraction of u_H^* on the right of $\tilde{u}$ becomes part of the basin of attraction of u_L^* and a piece of the basin of attraction of u_L^* on the left of $\tilde{u}$ becomes part of the basin of attraction of u_H^*. The opposite occurs for i even: that is, a piece of the basin of attraction of u_H^* on the left of $\tilde{u}$ becomes part of the basin of attraction of u_L^* and a piece of the basin of attraction of u_L^* on the right of $\tilde{u}$ becomes part of the basin of attraction of u_H^*.

At each stage, the basins of attraction of u_L^* and u_H^* (or of other attractors with different periodicity or aperiodic) correspond respectively to

$$\bigcup_{i(e)=0}^{n} \left(\psi_j^{-(i(e)+1)}, \psi_j^{-i(e)}\right) \cup \bigcup_{i(o)=1}^{n+1} \left(\psi_k^{-i(o)}, \psi_k^{-(i(k)+1)}\right)$$

$$\bigcup_{i(e)=0}^{n}\left(\psi_k^{-i(e)},\psi_k^{-(i(e)+1)}\right)\cup\bigcup_{i(o)=1}^{n+1}\left(\psi_j^{-(i(o)+1)},\psi_j^{-i(o)}\right)$$

where $i(e)=0,2,\dots n$, $i(o)=1,3,\dots n+1$, $j=l$ when the preiterate is odd and $j=r$ when the preiterate is even and $k=l$ when the preiterate is even and $k=r$ when the preiterate is odd.

The process continues until θ is such that $\psi(0)=1$ or, by symmetry, such that $\psi(1)=0$.

When $\psi(0)>1$ $(\psi(1)<0)$, a repelling period 2 cycle $(1-\upsilon,\upsilon)$ enters the (0, 1) interval. Increasing θ this repelling period 2-cycle shrinks faster than the admissible interval $(1-\underline{u},\underline{u})$, i.e. $\upsilon<\overline{u}$. If u_0 starts within the interval (υ_0,υ_1), the system converges to an economically meaningful (periodic or aperiodic) attractor, i.e., the inequalities $0<u\leq 1$ hold. It diverges to $\pm\infty$ otherwise.

Within the interval $(1-\upsilon,\upsilon)$, the basins of attractions of u_L^* and u_H^* are still highly disconnected and intermingled.

Increasing θ, when θ crosses θ^F, u_L^* and u_H^* lose simultaneously local stability. Two locally stable period two cycle emerge. Increasing further θ, cycles of any order and irregular cycles emerge according to a period doubling route to chaotic behaviour (See Figure A.5(a), plotted for $30\leq\theta\leq 60$ and $u_0=0.49$). Due to the symmetric properties of the map, two symmetric local attractors exist for $\theta^F<\theta<\theta^G$, the first cycling around u_L^* and the second around u_H^* (See Figure A.5(b) and A.5(c), plotted for $\theta=48$, $\gamma=0.4$ and with initial conditions $u_0=0.49$ and $u_0=0.51$, respectively).

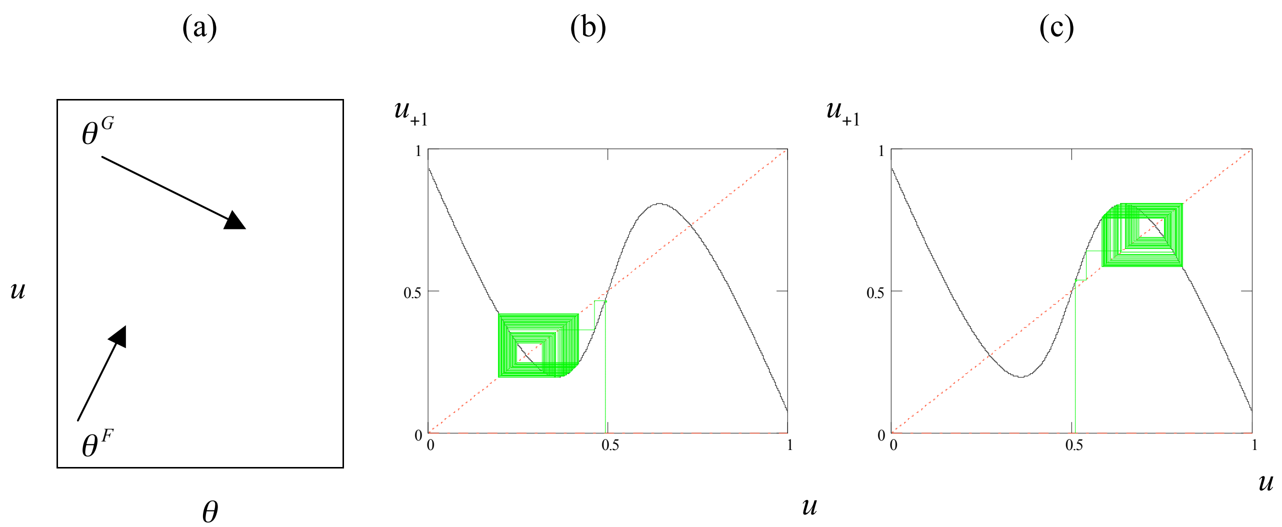


Figure A.5

Figures A.5(b) and A.5(c) show the existence of two trapping regions for two locally stable chaotic attractors. The first is at the left of $\tilde{u}$, corresponding to (c^m, c_1^m); the second is at the right of $\tilde{u}$, corresponding to (c_1^M, c^M). If one orbit starts within a trapping set, it can never escape from it.

At $\theta = \theta^G$, the system undergoes a substantial qualitative change. In correspondence of this value, $c_1^m = c_1^M = \tilde{u}$. The two locally stable chaotic attractors disappear and a new chaotic attractor emerges to which almost all trajectories converges. The trapping set of the new attractor is (c^m, c^M). See Figure A.6, plotted for $\theta = 57$.

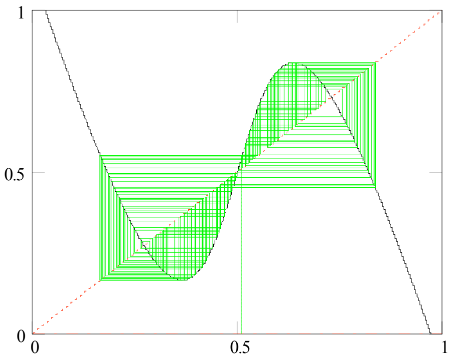


Figure A.6

The global bifurcation occurring at $\theta = \theta^G$ is less catastrophic compared to the one occurring for the Kaleckian case. Indeed, for example, the average value of the degree of capital utilisation for θ = 57 is 0.509 a value slightly larger than the normal degree of capacity utilisation, $\tilde{u} = 0.5$.

When $c^M > \upsilon$ almost all trajectories in the interval $(1-\upsilon, \upsilon)$ exit the (0, 1) interval and diverge to $\pm\infty$.

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